Accepted for publication. DOI: 10.1109/IEEE-IWS.2018.8400872

Power Maximization for Capacitive Wireless Power Transfer with Two Transmitters and One Receiver

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***Abstract*—Inductive wireless power transfer from multiple transmitters to a single receiver has already been studied. However, for capacitive power transfer, an analysis of a multi transmitter setup has not yet been performed, even though useful applications are being developed for this technology. In this work, we determine the optimal solution for a capacitive wireless power transfer system in order to maximize the power transfer from two transmitter sources to a single load at the receiver side. As well the case for uncoupled as coupled transmitters is solved. We find that for the uncoupled configuration, a purely resistive load optimizes the system, whereas for the coupled configuration, a susceptance has to be inserted to maximize the power transfer. We confirm our results by circuit simulations.**

I. Introduction

By means of the electric field, capacitive coupling allows wireless energy transfer. Applications are for example the charging of integrated circuits, biomedical devices, low-power consumer applications and electric vehicles [1], [2]. These examples consist of a single transmitter device, supplied by a source, and a single receiver, containing the load. This configuration has already been described in detail [1], [3], [4].

For certain applications, a configuration with multiple transmitters and/or multiple receivers could be beneficial. For example, a large transmitter plate can charge multiple receivers at once. But also the configuration with multiple transmitters and one receiver can be of interest. An example is the charging of electric vehicles while driving: multiple transmitter plates in the road could charge a driving vehicle which contains the receiver plate [1], [5], [6]. To our knowledge, a set-up with multiple transmitters for capacitive wireless power transfer (CPT) has not yet been described.

In this work, we provide the optimal solution to maximize the power transfer from *two* transmitter sources to a *single* load at the receiver. This problem has already been solved for *one* transmitter and *one* receiver [4], and for *one* transmitter and *two* receivers [7], but not yet for a multiple transmitter CPT set-up. As well the case with uncoupled as with coupled transmitters is solved. The results are confirmed by circuit simulations in SPICE.

II. Analytical solution

A. Circuit analysis

Fig. 1 represents a general CPT system with two transmitters and one receiver [1], [8]. We only consider the wireless link itself and do not take into account the remote electronics. The transmitters are supplied by the sinusoidal current sources, represented by the peak current phasors I_2 and I_3. We assume both transmitters operate at the same angular frequency ω_0. The conductances g_{11}, g_{22} and g_{33} represent the resistive losses in the circuit. The load of the receiver is given by $Y_1 = G_1 + jB_1$, with G_1 and B_1 the load conductance and susceptance, respectively.

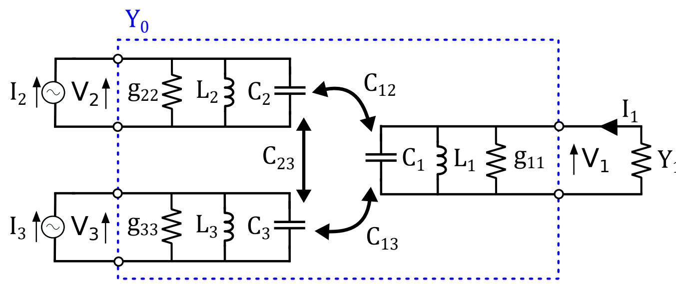

Fig. 1. A CPT circuit with two transmitters and one receiver can be represented by a three-port network, characterized by its admittance matrix $\boldsymbol{Y_0}$, indicated by the dashed rectangle.

The capacitive wireless link can be represented by coupled capacitances C_1, C_2 and C_3 [7], [8]. Power transfer is realized from the transmitter capacitances C_2 and C_3 to the receiver capacitance C_1, expressed by the mutual capacitances C_{12} and C_{13}, respectively. The coupling between the transmitter capacitances is given by the mutual capacitance C_{23}. The corresponding coupling factors k_{ij} (i,j=1,2,3) are defined by:

$$k_{ij} = \frac{C_{ij}}{\sqrt{C_i C_j}} \tag{1}$$

Resonance is created in each circuit by the inductors L_i (i=1,2,3) in parallel, with a value of

$$L_i = \frac{1}{\omega_0^2 C_i} \tag{2}$$

In this work, we will provide the analytical solution for power maximization: we will determine the optimal load Y_1 to maximize the output power P_1 delivered to the load.

For convenience, we introduce the following notations (i, j=1,2,3):

$$x_{ij} = \omega_0 C_{ij} \quad (3)$$

$$\chi_{ij} = \frac{x_{ij}}{\sqrt{g_{ii} g_{jj}}} \quad (4)$$

$$\theta = \sqrt{1 + \chi_{12}^2 + \chi_{13}^2} \quad (5)$$

We consider the three-port network as defined in Fig. 1. V_i and I_i (i=1,2,3) are the peak voltage and peak current phasors, respectively. The three-port network is fully characterized by its admittance matrix $\boldsymbol{Y_0}$ at frequency ω_0. We can express the current-voltage relation of the three-port as $\boldsymbol{I} = \boldsymbol{Y_0}.\boldsymbol{V}$, or:

$$\begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \begin{bmatrix} g_{11} & -jx_{12} & -jx_{13} \\ -jx_{12} & g_{22} & -jx_{23} \\ -jx_{13} & -jx_{23} & g_{33} \end{bmatrix} . \begin{bmatrix} V_1 \\ V_2 \\ V_3 \end{bmatrix} \quad (6)$$

B. Uncoupled transmitters

We first consider the case where the transmitters are uncoupled, i.e. $x_{23} = 0$. For this configuration, both transmitters function independently, and the entire system can be considered as the superposition of two separate CPT systems, each with one transmitter and one receiver. For this configuration, it was shown in [4] that the optimal load for power maximization is purely real, i.e. $B_1 = 0$. We thus obtain, with $I_1 = -G_1 V_1$:

$$\begin{bmatrix} 0 \\ I_2 \\ I_3 \end{bmatrix} = \begin{bmatrix} g_{11} + G_1 & -jx_{12} & -jx_{13} \\ -jx_{12} & g_{22} & 0 \\ -jx_{13} & 0 & g_{33} \end{bmatrix} . \begin{bmatrix} V_1 \\ V_2 \\ V_3 \end{bmatrix} \quad (7)$$

Inverting the 3x3 matrix, we find:

$$V_1 = \frac{g_{22} x_{13} I_3 + g_{33} x_{12} I_2}{D} j \quad (8)$$

with

$$D = g_{11} g_{22} g_{33} + G_1 g_{22} g_{33} + g_{22} x_{13}^2 + g_{33} x_{12}^2 \quad (9)$$

The output power P_1 is given by [9]:

$$P_1 = \frac{1}{2} G_1 |V_1|^2 \quad (10)$$

Substituting (8) in (10) and equalizing the partial derivative of (10) with regard to G_1 to zero, we find the optimal value for G_1 that maximizes power transfer:

$$G_{1,opt} = g_{11} \theta^2 \quad (11)$$

With this optimal load, the output power P_1 results in:

$$P_{1,max} = \frac{1}{8\theta^2} \left[\frac{\chi_{12} I_2}{\sqrt{g_{22}}} + \frac{\chi_{13} I_3}{\sqrt{g_{33}}} \right]^2 \quad (12)$$

We notice that, if only one transmitter would be present (e.g, $x_{13} = \chi_{13} = 0$), we obtain the same value for the optimal load and output power as derived by [4] for the configuration with a single transmitter and a single receiver.

C. Coupled transmitters

We now consider the case where the transmitters are coupled ($x_{23} \neq 0$). We determine the Thévenin equivalent as seen from port 1. The equivalent Thévenin voltage V_{th} can be found by calculating the open circuit voltage at port 1, and thus by solving the system of equations given by (6) with $I_1 = 0$ and $V_1 = V_{th}$. We obtain

$$V_{th} = -\frac{x_{23}(x_{12} I_3 + x_{13} I_2) - j(g_{22} x_{13} I_3 + g_{33} x_{12} I_2)}{g_{11} g_{22} g_{33} (\theta^2 + \chi_{23}^2) + 2j x_{12} x_{13} x_{23}} \quad (13)$$

The equivalent Thévenin admittance Y_{th} can be computed by $Y_{th} = I_1/V_1$ for the network where the current sources are replaced by an open circuit. By solving the system of equations given by (6) with $I_2 = I_3 = 0$, we obtain:

$$Y_{th} = g_1 \frac{\theta^2 + \chi_{23}^2}{1 + \chi_{23}^2} + j \frac{2 x_{12} x_{13} x_{23}}{g_{22} g_{33} + x_{23}^2} \quad (14)$$

According to the maximum power transfer theorem, the optimal load $Y_{1,opt} = G_{1,opt} + jB_{1,opt}$ at port 1 that realizes maximum power transfer is the complex conjugate of Y_{th}. We obtain

$$G_{1,opt} = g_{11} \frac{\theta^2 + \chi_{23}^2}{1 + \chi_{23}^2} \quad (15)$$

$$B_{1,opt} = -2 g_{11} \frac{\chi_{12} \chi_{13} \chi_{23}}{1 + \chi_{23}^2} \quad (16)$$

Notice that, contrary to the uncoupled configuration, a non-zero load susceptance is now necessary to realize power maximization.

From the Thévenin circuit loaded with $Y_{1,opt}$, the maximum output power $P_{1,max}$ can be calculated straightforwardly:

$$P_{1,max} = (\chi_{23}^2 + \theta^2) . \frac{(\sqrt{g_{22}} \chi_{13} I_3 + \sqrt{g_{33}} \chi_{12} I_2)^2 + \chi_{23}^2 (\sqrt{g_{22}} \chi_{12} I_3 + \sqrt{g_{33}} \chi_{13} I_2)^2}{8 g_{22} g_{33} (1 + \chi_{23}^2)[(\chi_{23}^2 + \theta^2)^2 + 4 \chi_{12}^2 \chi_{13}^2 \chi_{23}^2]} \quad (17)$$

For $\chi_{23} = 0$, (15), (16) and (17) reduce to the optimal load and output power for the uncoupled configuration.

We want to emphasize the similarity of our results with the dual configuration for inductive wireless power transfer [10].

III. Numerical validation

We confirm the analytical derivation by circuit simulation in SPICE. Consider the network with two transmitters and one receiver of Fig. 1. We simulate two scenarios: the uncoupled and the coupled configuration, with the values given in Table I. Both scenarios are identical, except for the coupling k_{23} between the transmitters.

For the uncoupled scenario, calculation from (11) results in the optimal load $G'_{1,opt}$ for achieving maximum power transfer. Remember that the optimal load for the uncoupled configuration is purely resistive. We obtain $G'_{1,opt} = 1.6$ mS. The corresponding output power $P'_{1,max}$ can be computed from (12): $P'_{1,max}$= 1.9 W.

TABLE I

SIMULATION VALUES FOR THE UNCOUPLED AND COUPLED CONFIGURATION. BOTH SCENARIOS ARE IDENTICAL, EXCEPT FOR THE COUPLING k_{23} BETWEEN THE TRANSMITTERS.

	Uncoupled	Coupled
f	1.0 MHz	1.0 MHz
I_2	200 mA	200 mA
I_3	100 mA	100 mA
g_{11}	1.0 mS	1.0 mS
g_{22}	1.0 mS	1.0 mS
g_{33}	2.0 mS	2.0 mS
C_1, C_2, C_3	400 nF	400 nF
L_1, L_2, L_3	63.3 μH	63.3 μH
k_{12}	25%	25%
k_{13}	25%	25%
k_{23}	0%	50%

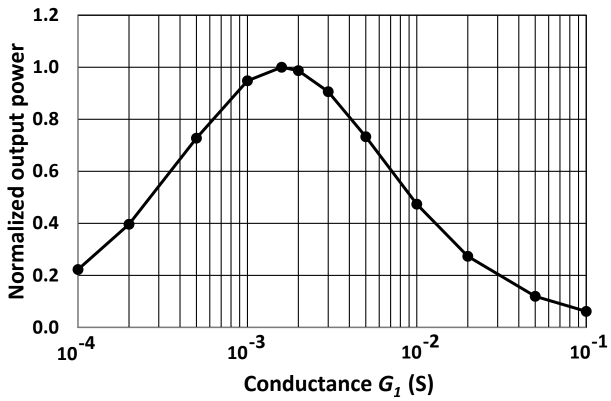

Fig. 2. The simulated output power, normalized to $P'_{1,max}$ =1.9 W, for varying load conductance for the uncoupled scenario.

We first simulate the uncoupled scenario in SPICE for varying load conductance. Fig 2 shows the output power, normalized to $P'_{1,max}$. Notice that as well the value for the optimal load as for the output power correspond perfectly with the analytical derivation.

Secondly, consider the coupled scenario (Table I). The optimal load $G''_{1,opt} + jB''_{1,opt}$ now has a non-zero susceptance. We obtain $G''_{1,opt}$= 1.3 mS and $B''_{1,opt}$= -0.28 mS from (15) and (16). The negative susceptance corresponds with an inductance of 574 μH. From (17), the maximum output power $P''_{1,max}$ can be calculated: $P''_{1,max} =$ 1.1 W. Fig. 3 shows the simulation results for the output power, normalized to $P''_{1,max}$. The simulation results validate the analytical expressions.

Finally, we simulate the coupled scenario for varying load susceptance. Fig. 4 shows the output power, normalized to $P''_{1,max}$, as function of varying load inductance. The load conductance is kept fixed at $G''_{1,opt} =$ 1.3 mS. The optimal load inductance corresponds with the value that was analytically derived: 574 μH. Notice that for this example, the output power remains high for inductances larger than $G''_{1,opt}$.

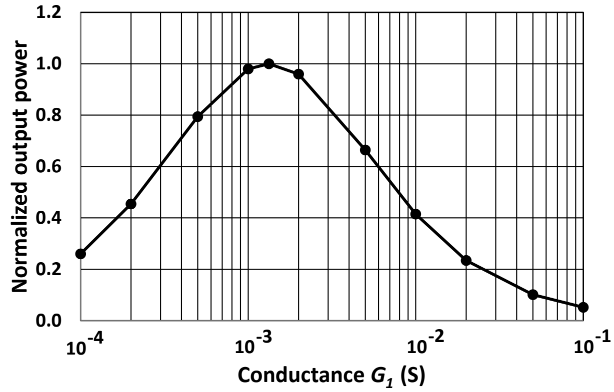

Fig. 3. The simulated output power, normalized to $P''_{1,max}$ =1.1 W, for varying load conductance for the coupled scenario. The load susceptance remains invariant and corresponds with an inductance of 574 μH.

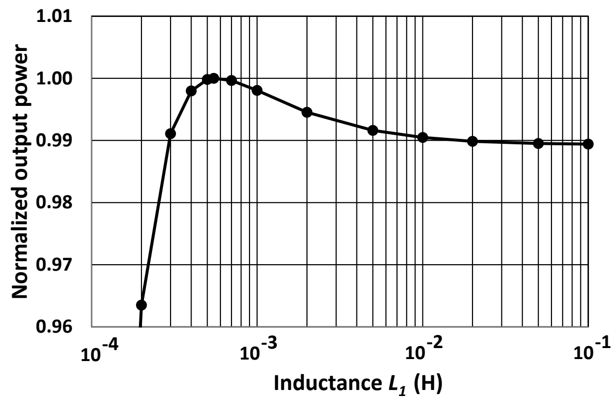

Fig. 4. The simulated output power, normalized to $P''_{1,max}$ =1.1 W, for varying load inductance for the coupled scenario. The load conductance remains fixed at $G''_{1,opt}$ = 1.3 mS.

IV. CONCLUSION

We analytically solved the power maximization problem for a CPT setup with two transmitters and a single receiver. If both transmitters are uncoupled, a purely resistive load applies. For coupled transmitters, a non-zero load susceptance is necessary for power maximization. The analytical expressions were numerically validated by SPICE simulations.

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