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Optimization of a Capacitive Wireless Power Transfer System with Two Electric Field Repeaters

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Summary

In this paper, a capacitive wireless power transfer link with two repeaters between the transmitter and receiver is analyzed. The performance of the system is optimized by maximizing the different gains. Two approaches of practical interest are analytically studied and solved. First, the couplings between different resonators is considered fixed and the optimal load and source of the system are determined as function of the network characteristics. Second, the performance is optimized by varying the couplings between resonators for a configuration with a given (non-ideal) source and load. The analytical results are verified through circuital simulations. It is demonstrated that, under certain conditions, both approaches allow for the maximization of the different gains, resulting in more optimization options than a single repeater configuration.

KEYWORDS:
capacitive wireless power transfer, resonator, repeater, relay, gains

1 | INTRODUCTION

Capacitive wireless power transfer (CPT) is an upcoming technology enabling the wireless charging and powering of mobile devices[1,2]. In contrary to the more technologically mature inductive power transfer (IPT), that uses the *magnetic* field to transport energy, CPT applies the *electric* field. One of the main advantages of CPT compared to IPT are the cheap and low-cost coupling elements: instead of bulky and expensive coils, necessary for IPT, CPT applies simple conductive plates.

Unfortunately, CPT for long distances suffers from low power density due to a low coupling capacitance[1]. In order to compensate for this disadvantage, inspiration can be found in IPT: the use of intermediate coils (also called relays or magnetic field repeaters) between the transmitter and receiver coil allow for[3,4,5,6]

- extra degrees of freedom to maximize the system performance.
- efficient impedance matching by highly coupled transmitter-relay or relay-receiver links.

It has been found[3,5,7] that two relays (i.e. four coils) for an IPT system perform better with regard to performance versus distance than none or a single relay.

Similarly as to relays in IPT, intermediate coupling plates between the transmitter and receiver plates of a CPT setup can allow better system performance for wireless power transfer at greater distances. These intermediate coupling circuits are referred to as electric field repeaters[1,4]. Figure 1 shows a schematic overview of a CPT system with two repeaters. Wireless power transfer is realized from a transmitter source I_S to a load Y_L via two intermediate electric field repeaters. Compensation circuits are present in each of the four physically separated circuits in order to create resonance.

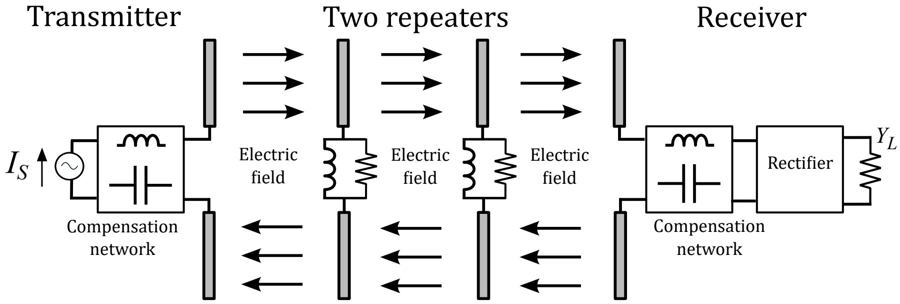

Figure 1 Schematic overview of a capacitive wireless power transfer system with two repeaters.

In this work, it will be demonstrated how the repeater configuration can be applied to increase system performance. However, other applications of the repeater topology are possible, e.g.:

- Via one or more intermediate elements, it is possible to increase the total power transfer distance [4,5], allowing for example to improve the convenience and user experience of powering biomedical implants [8].
- The use of multiple repeaters can allow for a change in the spatial power flow [9].
- The repeater configuration can also be used to power different loads simultaneously over larger distances [10]. A practical application could be the charging of stacked reefer containers [11].

The performance of a CPT system can be described by the standard gain expressions (power gain, available gain and transducer gain). Depending on the application, the maximization of one of the three gains is relevant [12,13]. For example, the transducer gain is relevant when one wants to maximize the amount of power delivered to the load, whereas the power and available gain are of interest if the goal is to minimize the power reflections at output or input port, respectively.

In this work, we determine the requirements of a CPT system with 2 repeaters for maximizing each of the three main gains. More specifically, two problems are solved:

- **Optimal terminations solution** – The optimal terminations of the system at input and output port for each gain are determined, i.e., the load value and the internal resistance of the source. All other components of the CPT system are considered fixed and given, including all the couplings.
- **Optimal coupling solution**– The optimal coupling between transmitter and first repeater on the one hand, and between the second repeater and receiver on the other hand are determined. All the other components of the system are fixed, including the terminations of the system. Note that it is assumed that the coupling between the two repeaters is given. In this way, this coupling can realize the distance requirement for the wireless power transfer.

In a practical realization, communication between transmitter and receiver can be necessary. The aforementioned optimizations have already been done for as well four-coil IPT [7,14] as for CPT with a single repeater [12,15], but as far as we know, no solution for a CPT system with two repeaters has yet been presented. Relying upon the duality principle between IPT and CPT [16,17], the same network formalism methodology as [7,14] is exploited.

The structure of this work is the following: first, the equivalent circuit of a CPT system with two electric field repeaters (Section 2) is introduced. Next, the optimal terminations are analytically determined in Section 3 by applying the conjugate image theory. In Section 4, the optimal couplings are derived for maximizing the different gains. Finally, the analytical derivation is validated by simulating a numerical example via an electric circuit simulator (Section 5).

2 | CPT WITH TWO REPEATERS

2.1 | Equivalent circuit

The CPT system with two repeaters can be represented by the equivalent circuit of Fig. 2 [12]. The subscripts 1 and 2 indicate the transmitter and receiver, respectively. The subscripts 3 and 4 are used for the two repeaters.

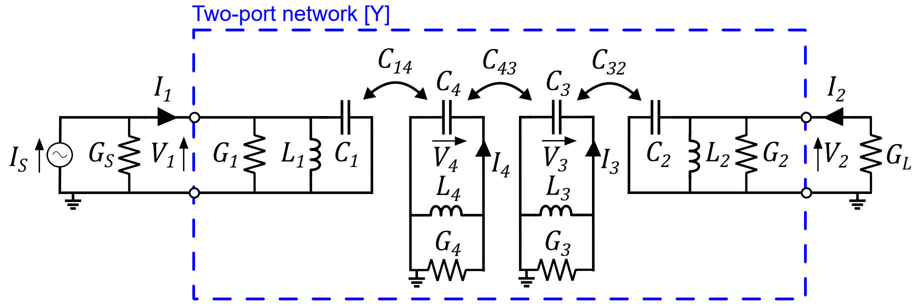

Figure 2 Equivalent circuit of a capacitive wireless power transfer system with two repeaters (subscripts 3 and 4) between the transmitter (subscript 1) and the receiver (subscript 2).

The supply is represented by the current source I_S with internal shunt conductance G_S and angular frequency ω_0. It is assumed that the power available from the generator is constant. As a consequence, if G_S is varied, the current I_S is varied accordingly. The goal of the system is to transfer energy to a load, represented by the conductance G_L. Resonance is created in each circuit by applying shunt inductors with value:

$$L_i = \frac{1}{\omega_0^2 C_i}. \tag{1}$$

In this way, the four resonator circuits 1, 2, 3 and 4 are constructed. The resistive losses are given by the conductances G_i (i=1,2,3,4). Note that the series resistance of the inductors is neglected.

The electric couplings between the transmitter, repeaters and receiver can be expressed by the mutual capacitances C_{ij} (i, j=1,2,3,4; $i \neq j$) between the capacitances C_i and C_j, which are related to the coupling factor k_{ij} given by:

$$k_{ij} = \frac{C_{ij}}{\sqrt{C_i C_j}}. \tag{2}$$

The values of C_i do not correspond to the capacitances physically present in the CPT system, but represent an equivalent circuit representation [18].

It is assumed that there is no coupling between non-neighboring circuits ($k_{12} = k_{14} = k_{23} = 0$). They can be neglected compared to the coupling of the neighboring circuits, due to the larger distance.

2.2 | Admittance matrix

Taken into account the peak voltage phasors V_i and peak current phasors I_i (i=1,2,3,4) as defined in Fig. 2, Kirchhoff's current laws result into the following:

$$I_1 = G_1 V_1 + j\omega_0 C_{14} V_4 \tag{3}$$

$$I_2 = G_2 V_2 + j\omega_0 C_{32} V_3 \tag{4}$$

$$0 = G_3 V_3 + j\omega_0 C_{32} V_2 + j\omega_0 C_{43} V_4 \tag{5}$$

$$0 = G_4 V_4 + j\omega_0 C_{14} V_1 + j\omega_0 C_{43} V_3. \tag{6}$$

The system can be modeled as a two port network (Fig. 2), characterized by an admittance matrix $\boldsymbol{Y}$ with elements y_{ij}:

$$\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} y_{11} & y_{12} \\ y_{21} & y_{22} \end{bmatrix} \cdot \begin{bmatrix} V_1 \\ V_2 \end{bmatrix}. \tag{7}$$

From (3), (4), (5) and (6), the admittance matrix elements result into:

$$y_{11} = \frac{G_1 G_3 G_4 + \omega_0^2 G_1 C_{43}^2 + \omega_0^2 G_3 C_{14}^2}{G_3 G_4 + \omega_0^2 C_{43}^2} \tag{8}$$

Table 1 Overview of the different parameters (i, j=1,2,3,4; $i \neq j$).

$k_{ij} = \frac{C_{ij}}{\sqrt{C_i C_j}}$	$B_0 = \omega_0 C_2$	$\chi_{ij} = \sqrt{Q_i Q_j} k_{ij}$
$Q_i = \frac{\omega_0 C_i}{G_i}$	$n_{12} = \sqrt{\frac{C_1}{C_2}}$	$\rho = 1 + \chi_{43}^2$
$Q_S = \frac{\omega_0 C_1}{G_S}$	$Q_{1T} = \frac{Q_1 Q_S}{Q_1 + Q_S}$	$\chi_{1T4} = \sqrt{Q_{1T} Q_4} k_{14}$
$Q_L = \frac{\omega_0 C_2}{G_L}$	$Q_{2T} = \frac{Q_2 Q_L}{Q_2 + Q_L}$	$\chi_{32T} = \sqrt{Q_3 Q_{2T}} k_{32}$

$$y_{12} = y_{21} = -j \frac{\omega_0^3 C_{14} C_{43} C_{32}}{G_3 G_4 + \omega_0^2 C_{43}^2} \tag{9}$$

$$y_{22} = \frac{G_2 G_3 G_4 + \omega_0^2 G_2 C_{43}^2 + \omega_0^2 G_4 C_{32}^2}{G_3 G_4 + \omega_0^2 C_{43}^2}. \tag{10}$$

Note that the diagonal elements are purely real and the non-diagonal elements are purely imaginary.

The unloaded quality factors Q_i are introduced:

$$Q_i = \frac{\omega_0 C_i}{G_i}. \tag{11}$$

As normalization parameter, the susceptance B_0 of the receiver is used:

$$B_0 = \omega_0 C_2. \tag{12}$$

In order to alleviate the notation, the parameters χ_{ij}, ρ and the transformation ratio n_{12} are defined:

$$\chi_{ij} = \sqrt{Q_i Q_j} k_{ij} \tag{13}$$

$$\rho = 1 + \chi_{43}^2 \tag{14}$$

$$n_{12} = \sqrt{\frac{C_1}{C_2}}. \tag{15}$$

An overview of the parameters can be found in Table 1.

Rewriting the admittance matrix elements as function of the aforementioned definitions results into:

$$y_{11} = \frac{B_0 n_{12}^2}{\rho Q_1} (\rho + \chi_{14}^2) \tag{16}$$

$$y_{12} = y_{21} = -j \frac{B_0 n_{12}}{\rho \sqrt{Q_1 Q_2}} \chi_{14} \chi_{43} \chi_{32} \tag{17}$$

$$y_{22} = \frac{B_0}{\rho Q_2} (\rho + \chi_{32}^2). \tag{18}$$

The input admittance Y_{in} and output admittance Y_{out} of the two port network are given by [12]:

$$Y_{in} = G_{in} + j.B_{in} = y_{11} - \frac{y_{12} y_{21}}{y_{22} + Y_L}. \tag{19}$$

$$Y_{out} = G_{out} + j.B_{out} = y_{22} - \frac{y_{12} y_{21}}{y_{11} + Y_S}. \tag{20}$$

Substituting the admittance matrix elements (16), (17) and (18) into these expressions results into the input and output conductance:

$$G_{in} = \frac{B_0 n_{12}^2}{Q_1} \frac{\left(\chi_{14}^2 + 1\right)\left(\chi_{32T}^2 + 1\right) + \chi_{43}^2}{\rho + \chi_{32T}^2}, \tag{21}$$

$$G_{out} = \frac{B_0}{Q_2} \frac{\left(\chi_{1T4}^2 + 1\right)\left(\chi_{32}^2 + 1\right) + \chi_{43}^2}{\rho + \chi_{1T4}^2}. \tag{22}$$

Note that the input and output admittance are real, since the internal shunt admittance and load are considered conductances.

2.3 | Gains

The three power gains (i.e., power, available and transducer gain) will be used as figures of merit of the performance of the network. The reader is referred to [12,13,19] for an in-depth discussion. The power gains for the analyzed network can be easily derived starting from the general expressions for a two–port network described by its admittance matrix [12].

The power gain G_P is defined as the ratio between the power P_L dissipated in the load and the active power P_{in} into the two-port network. It corresponds to the power conversion efficiency in the context of wireless power transfer. As function of the elements of the admittance matrix, it is given by:

$$G_P = \frac{P_L}{P_{in}} = \frac{G_L}{G_{in}} \left| \frac{y_{21}}{y_{22} + Y_L} \right|^2. \tag{23}$$

Next, the available gain G_A is given by the ratio between the maximum available load power P_A and the input power P_{AG} available form the generator. It equals:

$$G_A = \frac{P_A}{P_{AG}} = \frac{G_S}{G_{out}} \left| \frac{y_{21}}{y_{11} + Y_S} \right|^2. \tag{24}$$

Finally, the transducer gain G_T is a measure for the amount of power that is delivered to a load for a fixed available input power of the generator. It is given by:

$$G_T = \frac{P_L}{P_{AG}} = 4 G_L G_S \left| \frac{y_{21}}{(y_{11} + Y_S)(y_{22} + Y_L) - y_{12} y_{21}} \right|^2. \tag{25}$$

Substituting the admittance matrix elements (16), (17) and (18) into the expressions for the gains (23), (24) and (25), the different gains for a CPT system with 2 repeaters can be obtained:

$$G_P = \frac{\chi_{14}^2 \chi_{43}^2 \chi_{32T}^2}{\left[\left(\chi_{14}^2 + 1\right)\left(\chi_{32T}^2 + 1\right) + \chi_{43}^2\right]\left(\rho + \chi_{32T}^2\right)} \frac{Q_{2T}}{Q_L}, \tag{26}$$

$$G_A = \frac{Q_{1T}}{Q_S} \frac{\chi_{1T4}^2 \chi_{43}^2 \chi_{32}^2}{\left[\left(\chi_{1T4}^2 + 1\right)\left(\chi_{32}^2 + 1\right) + \chi_{43}^2\right]\left(\rho + \chi_{1T4}^2\right)}, \tag{27}$$

$$G_T = \frac{Q_{1T}}{Q_S} \frac{4 \chi_{1T4}^2 \chi_{43}^2 \chi_{32T}^2}{\left[\left(\chi_{1T4}^2 + 1\right)\left(\chi_{32T}^2 + 1\right) + \chi_{43}^2\right]^2} \frac{Q_{2T}}{Q_L}. \tag{28}$$

3 | OPTIMAL TERMINATIONS SOLUTION

It is assumed that all components and couplings of the CPT system are given, i.e., the admittance matrix is fixed. The question arises what the optimal terminations (i.e., the internal resistance of the source and the load) at the input and the output port of the two-port network are in order to optimize the different gains.

It is well-known that in order to maximize the gains, the conjugate matching conditions are relevant. It has been shown [20] that for a general two-port network, the terminating admittances which minimize power reflections are the conjugate image admittances $Y_{ci} = G_{ci} + jB_{ci}$ (i=1,2) of the network. For a general two-port network characterized by admittance matrix elements $y_{ij} = g_{ij} + jb_{ij}$, they are complex quantities, given by [12,20]:

$$Y_{c1} = G_{c1} + jB_{c1} = g_{11}(\theta_g + j\theta_b) - jb_{11} \tag{29}$$

$$Y_{c2} = G_{c2} + jB_{c2} = g_{22}(\theta_g + j\theta_b) - jb_{22} \tag{30}$$

with:

$$\theta_g = \sqrt{\left(1 - \frac{g_c^2}{g_{11}g_{22}}\right)\left(1 + \frac{b_c^2}{g_{11}g_{22}}\right)} \tag{31}$$

$$\theta_b = \frac{g_c b_c}{g_{11}g_{22}} \tag{32}$$

$$g_c + jb_c = \sqrt{y_{12}y_{21}}. \tag{33}$$

Substituting the admittance matrix elements (16), (17) and (18) into (29) and (30) results into the optimal terminations of a CPT system with 2 repeaters:

$$Y_{c1} = G_{c1} = \frac{B_0 n_{12}^2}{\rho Q_1}\sqrt{(\rho + \chi_{14}^2)^2 + \frac{\rho + \chi_{14}^2}{\rho + \chi_{32}^2}\chi_{14}^2\chi_{43}^2\chi_{32}^2} \tag{34}$$

$$Y_{c2} = G_{c2} = \frac{B_0}{\rho Q_2}\sqrt{(\rho + \chi_{32}^2)^2 + \frac{\rho + \chi_{32}^2}{\rho + \chi_{14}^2}\chi_{14}^2\chi_{43}^2\chi_{32}^2}. \tag{35}$$

One can see that the optimal terminations are purely resistive, i.e. they are image conductances: $Y_{c1} = G_{c1}$ and $Y_{c2} = G_{c2}$.

Once the conjugate image conductances are known, the maximization of the power and available gain is straightforward[12,13]:

- The power gain G_P is independent on the source conductance G_S and is maximized for a load conductance $G_L = G_{i2}$, given by (35).
- The available gain G_A is independent on the load G_L and is maximized for a source conductance $G_S = G_{i1}$, given by(34).

Regarding the transducer gain G_T, a distinction must be made depending on whether or not it is possible to vary both G_S and G_L. If only one conductance can be varied, the optimal termination equals the input or output conductance:

- If the source conductance G_S is given, a maximum G_T is achieved for $G_L = G_{out}$, given by (22). In this configuration, G_T equals G_A, given by (27).
- If the load G_L is given, a maximum G_T is achieved for $G_L = G_{in}$, given by (21). Now, G_T equals G_P, given by (26).

When both the source conductance G_S and load G_L can be varied, the optimal terminations equal the image admittances: $G_S = G_{i1}$ and $G_L = G_{i2}$. In this configuration, the three gains are equal ($G_P = G_A = G_T = G_M$). For the CPT system with two repeaters with optimal terminations, we obtain the ultimate gain G_M:

$$G_M = \frac{\left(\sqrt{(\rho + \chi_{14}^2)(\rho + \chi_{32}^2)} - \sqrt{\rho[(1 + \chi_{14}^2)(1 + \chi_{32}^2) + \chi_{43}^2]}\right)^2}{\chi_{14}^2\chi_{43}^2\chi_{32}^2} \tag{36}$$

4 | OPTIMAL COUPLING SOLUTION

In a practical setup, the source conductance G_S and load G_L are often predetermined by the application and not adjustable. However, it is possible to modify the couplings in order to vary the performance of the system. In this section, it is assumed that the terminations are fixed and that only the following parameters can be exploited for maximizing the gains:

- the coupling factor k_{14} between the transmitter and the first repeater.

- the coupling factor k_{32} between the second repeater and the receiver.

In order to meet the requirement for the power transfer distance, we assume that the coupling k_{43} between both repeaters is given and fixed.

4.1 | Optimal couplings for maximizing the power gain

First, the influence of the coupling k_{14} between the transmitter and the first repeater on the power gain is considered. Analysis of (26) shows that that higher k_{14}, the higher the power gain G_P. For high values of k_{14}, the power gain G_P is nearly independent on k_{14}.

Second, we examine the coupling k_{32} between the second repeater and the receiver. Deriving (26) to k_{32} and equating to zero results in the optimal value that maximizes the power gain G_P:

$$k_{32}^{GP} = \frac{1}{\sqrt{Q_3 Q_{2T}}} \sqrt[4]{\frac{\rho\left(\rho + \chi_{14}^2\right)}{\chi_{14}^2 + 1}}. \tag{37}$$

Since a coupling factor of more than one is physically impossible (i.e. $k_{32} < 1$), a solution only exists under the following condition:

$$Q_{2T} > \frac{1}{Q_3} \sqrt{\frac{\rho\left(\rho + \chi_{14}^2\right)}{\chi_{14}^2 + 1}}. \tag{38}$$

For the optimal value $k_{32} = k_{32}^{GP}$, the power gain results into:

$$G_{P32}^M = \frac{\left[\sqrt{\rho\left(\chi_{14}^2 + 1\right)} - \sqrt{\rho + \chi_{14}^2}\right]^2}{\chi_{14}^2 \chi_{43}^2} \frac{Q_{2T}}{Q_L}. \tag{39}$$

In this expression, the first factor equals the ultimate gain G_M of a CPT setup consisting only of the resonator circuits 1, 4 and 3, while the second factor represents the efficiency of the receiver circuit[7,13]. As a result, it can be deduced that the link between the second repeater and the receiver (i.e., k_{32}) is used as a matching network to transform the load into a reflected conductance equal to the image conductance of the circuit formed by the first three resonators.

4.2 | Optimal couplings for maximizing the available gain

The influence of the couplings on the available gain G_A is identical as the influence on G_P, with k_{14} and k_{32} interchanged. This is expected given the symmetry of G_A and G_P with respect to the input and output port.

The available gain G_A rises monotonically with k_{32}, being nearly independent of k_{32} at high couplings.

Equating the derivative of (27) to zero results in the optimal value for k_{14}:

$$k_{14}^{GA} = \frac{1}{\sqrt{Q_{1T} Q_4}} \sqrt[4]{\frac{\rho\left(\rho + \chi_{32}^2\right)}{\chi_{32}^2 + 1}}. \tag{40}$$

The condition $k_{14} < 1$ results into:

$$Q_{1T} > \frac{1}{Q_4} \sqrt{\frac{\rho\left(\rho + \chi_{32}^2\right)}{\chi_{32}^2 + 1}}. \tag{41}$$

At this optimum point, the available gain is given by:

$$G_{A14}^M = \frac{Q_{1T}}{Q_S} \frac{\left[\sqrt{\rho\left(\chi_{32}^2 + 1\right)} - \sqrt{\rho + \chi_{32}^2}\right]^2}{\chi_{43}^2 \chi_{32}^2}. \tag{42}$$

Similarly as for the power gain, the first factor equals the efficiency of the transmitter circuit, while the second factor represents the ultimate gain G_M of a CPT setup consisting only of the resonator circuits 4, 3 and 2[7,13]. Now, the link between the transmitter and the first repeater (i.e., k_{14}) is used as a matching network to transform the source losses into a reflected admittance equal to the image conductance of the circuit formed by the last three resonators.

4.3 | Optimal couplings for maximizing the transducer gain

Derivation of (28) to k_{14} and k_{32}, respectively, and equating to zero results into the following optimal couplings with respect to the transducer gain G_T:

$$k_{14}^{GT} = \frac{\sqrt[4]{\rho}}{\sqrt{Q_{1T} Q_4}}, \tag{43}$$

$$k_{32}^{GT} = \frac{\sqrt[4]{\rho}}{\sqrt{Q_3 Q_{2T}}}. \tag{44}$$

The requirements that $k_{14} < 1$ and $k_{32} < 1$ lead to the following conditions:

$$Q_{1T} > \frac{\sqrt{\rho}}{Q_4}, \tag{45}$$

$$Q_{2T} > \frac{\sqrt{\rho}}{Q_3}. \tag{46}$$

Note that the optimum k_{14}^{GT} coincides with k_{14}^{GA} in the limit of Q_2 to infinity. Analogously, k_{32}^{GT} and k_{32}^{GP} are equal if Q_1 goes to infinity.

At the optimal couplings k_{14}^{GT} and k_{32}^{GT}, the transducer gain G_T is maximized and equals:

$$G_T^M = \frac{Q_{1T}}{Q_S} \frac{\left(1 - \sqrt{\rho}\right)^2}{\chi_{43}^2} \frac{Q_{2T}}{Q_L}. \tag{47}$$

The second factor equals the ultimate gain G_M of a CPT setup consisting of only the repeater resonators circuits 3 and 4. The first and last factor represent the efficiency of the transmitter and receiver, respectively[7,13]. The wireless links k_{14} and k_{32} are now used as impedance matching tools to maximize the transducer gain and realize the conjugate image conductances at both input and output port.

It is relevant to note that, even when $k_{14} = k_{14}^{GT}$ and $k_{32} = k_{32}^{GT}$, the ultimate gain G_M is not achieved, i.e. the three gains are not equal. Indeed, calculating the input and output conductances of the input and output ports result into:

$$G_{in}^{GT} = G_S + 2G_1 \tag{48}$$

$$G_{out}^{GT} = G_L + 2G_2. \tag{49}$$

indicating that the input and output port are not matched to the source conductance G_S and load G_L, respectively. It can be seen that the losses in the transmitter and receiver circuit are the culprit. Even when G_T is maximized, the optimal values of G_P and G_A are not achieved. Only when the quality factors Q_1 and Q_2 limit infinity, the gains become equal and the transducer gain equals the ultimate gain of a CPT setup consisting of only the repeater resonators circuits:

$$\lim_{Q_1, Q_2 \to \infty} G_T^M = \frac{\left(1 - \sqrt{\rho}\right)^2}{\chi_{43}^2}. \tag{50}$$

However, we will show in the next section that in practical configurations, all three gains will be close to their maxima if k_{14} and k_{32} are optimized towards G_T.

5 | NUMERICAL VERIFICATION

In order to validate the analytical derivation, a numerical example is simulated by using the commercial simulator NI AWR Design Environment. We consider a CPT setup with two repeaters, characterized by the parameters listed in Table 2. The configuration topology corresponds to the equivalent circuit of Fig. 2. The system operates at a frequency f_0 of 10 MHz.

An overview of the calculated parameters can be found in Table 3. The shunt inductors are calculated from (1) in order to create resonance in each circuit at the operating frequency f_0. The coupling factors, quality factors, and transmission ratio are derived from (2), (11) and (15).

Table 2 Given parameters for the analyzed CPT system with two repeaters.

Quantity	Value	Quantity	Value
G_1	0.100 mS	C_1	300 pF
G_2	0.100 mS	C_2	250 pF
G_3	0.500 mS	C_3	350 pF
G_4	0.200 mS	C_4	300 pF
f_0	10.0 MHz	C_{43}	100 pF

Table 3 Calculated parameter values for the simulated CPT system with two repeaters.

Quantity	Value	Quantity	Value
L_1	0.844 μH	Q_1	188
L_2	1.013 μH	Q_2	157
L_3	0.724 μH	Q_3	44
L_4	0.844 μH	Q_4	94
B_0	15.7 mS	n_{12}	1.095
k_{43}	0.309		

First, the optimal terminations solution is verified by setting $G_S = G_{c1}$ and $G_L = G_{c2}$, which are calculated from (34) and (35). Circuital simulation results achieved by varying G_L for $R_S = 1/G_S = 1/G_{c1}$ =52.55 Ω (i.e., $G_S = G_{c1}$ =19.0 mS) are presented in Figure 3, while Figure 4 depicts the different gains for $R_L = 1/G_L = 1/G_{c2}$ =214.44 Ω, i.e. $G_L = G_{c2}$ =4.66 mS, and varying G_S. In both cases the values assumed for the gains are: k_{14}=0.5, k_{32}=0.338, k_{43}=0.309. It can be seen that G_P and G_A do not depend on G_S and G_L, respectively, while G_T is dependent on both. Additionally, it is clear from the simulations that all the three gains are maximized for the calculated image conductances $G_S = G_{c1}$ =19.0 mS and $G_L = G_{c2}$ =4.66 mS, attaining an ultimate gain G_M of 87.93%.

Second, the optimal coupling solution is considered by varying k_{14} and k_{32} for fixed (non-optimal) terminations. The different gains are plotted in Figures 5–7 for G_S =20.0 mS and G_L =10.0 mS. For these values, the optimal couplings can be calculated from (37), (40), (43) and (44). By fixing k_{14} at 0.4, from (37) it can be derived k_{32}^{GP} =0.557, resulting in G_{P32}^{M}=88.9%. Similarly, for the available gain, k_{14}^{GA} =0.512 if k_{32} is fixed at 0.4, resulting in G_{A14}^{M}=88.5%. The transducer gain is maximized for k_{14}^{GT} =0.474 and k_{32}^{GT} =0.539, resulting in G_T=89.1%. By observing Figures 5–7, it can be seen that the simulation results confirm the analytical derivation. Moreover, from Figure 7 it can be observed that G_T depends on both couplings. Regarding G_P, Figure 6 shows that for a given value of k_{32}, it is not possible to maximize G_P by varying k_{14}. Similarly, from Figure 5 is it clear that for a given value of k_{14}, it is not possible to maximize G_A by varying k_{32}.

In the previous section, it was shown that even when $k_{14} = k_{14}^{GT}$ and $k_{32} = k_{32}^{GT}$, the ultimate gain G_M is not achieved, i.e. the three gains are not equal, due to the losses in the transmitter and receiver circuit. However, when the quality factors Q_1 and Q_2 are high, which is often the case in a practical setup, all three gains will be close to their maxima. This is also reflected by the simulation, for which Q_1=189 and Q_2=157:

- When G_T is maximized (i.e., k_{14} =0.474 and k_{32} =0.539), the gains equal: $G_P = G_A = G_T$ =89.09% (see Figure 8),
- For k_{14} =0.474, G_P is maximized for k_{32}=0.552 and equals 89.10% (see Figure 9).
- For k_{32} =0.539, G_A is maximized for k_{14}=0.496 and equals 89.13% (see Figure 9).

It can be concluded that all the three gains are very close to their maxima for $k_{14} = k_{14}^{GT}$ and $k_{32} = k_{32}^{GT}$.

Note a significant difference in these results compared to a single repeater system[12,15]. If we consider a single repeater configuration with one coupling fixed in order to fulfill the requirement of power transfer distance, there is only one coupling left to

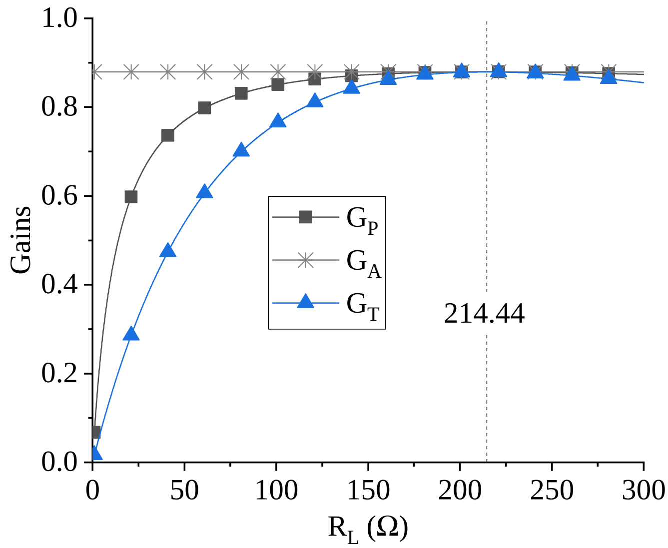

Figure 3 Results obtained by varying $R_L = 1/G_L$ for $R_S = 1/G_S = 1/G_{c1}$ =52.55 Ω, i.e. $G_S = G_{c1}$ =19.00 mS. The values assumed for the couplings are k_{14}=0.5, k_{32}=0.338, k_{43}=0.309.

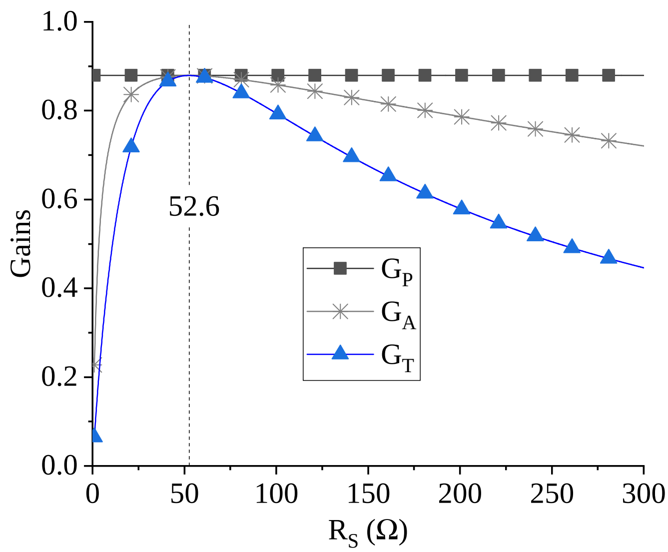

Figure 4 Results obtained by varying $R_S = 1/G_S$ for $R_L = 1/G_L = 1/G_{c2}$ =214.44 Ω, i.e. $G_L = G_{c2}$ =52.55 mS. The values assumed for the couplings are k_{14}=0.5, k_{32}=0.338, k_{43}=0.309.

be varied. As a result, it is *not* possible to maximize all three gains, contrary to the two repeater system that has an extra degree of freedom (i.e. two varying couplings). Indeed:

- If the coupling between transmitter and the repeater is varied to optimize the single repeater CPT setup, it is only possible to maximize G_A.
- If the coupling between repeater and receiver is varied as optimization parameter, it is only possible to maximize G_P.

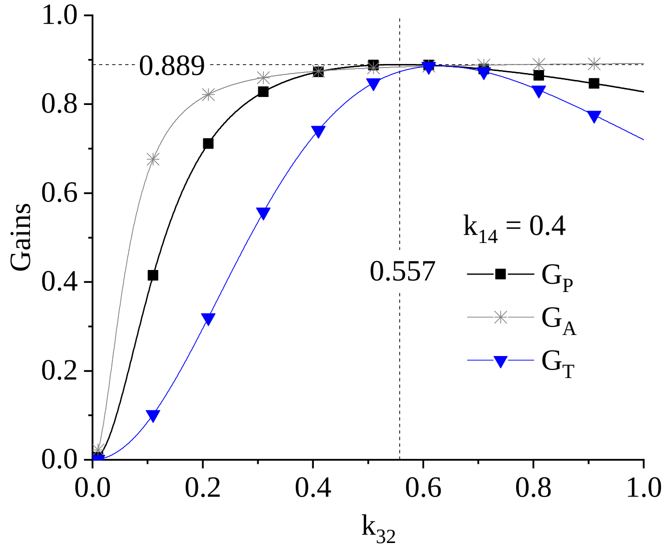

Figure 5 Results obtained by varying k_{32} for $k_{14} = 0.4$. Simulations have been performed assuming non-optimal values for the terminations: G_S =20.0 mS and G_L =10.0 mS.

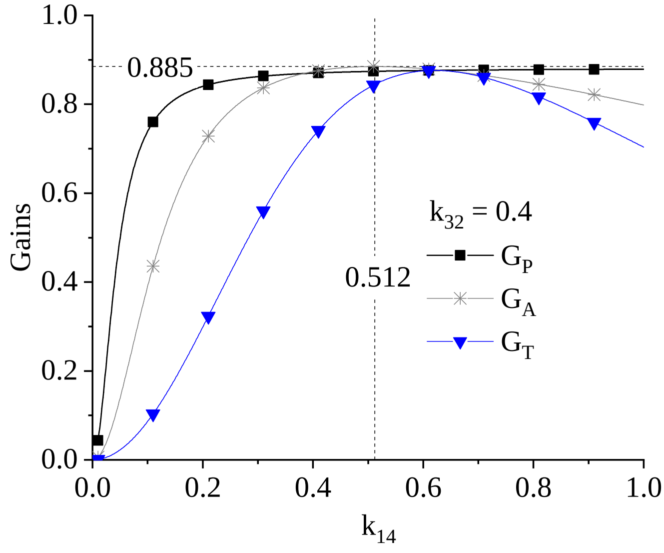

Figure 6 Results obtained by varying k_{14} for $k_{32} = 0.4$. Simulations have been performed assuming non-optimal values for the terminations: G_S =20.0 mS and G_L =10.0 mS.

Accordingly, only a local maximum for the transducer gain G_T and output power is achievable, analogous to an IPT configuration with a single intermediate coil [13].

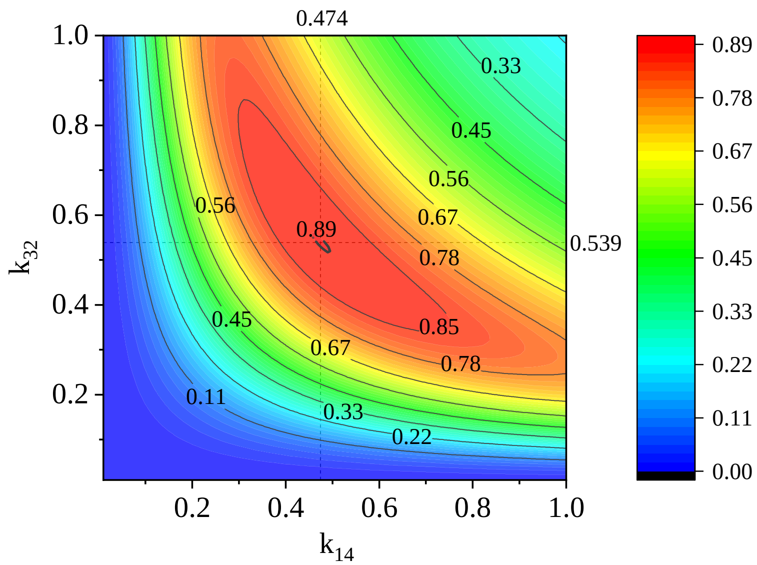

Figure 7 Results obtained for the transducer gain by varying k_{14} and k_{32} for G_S =20.0 mS and G_L =10.0 mS.

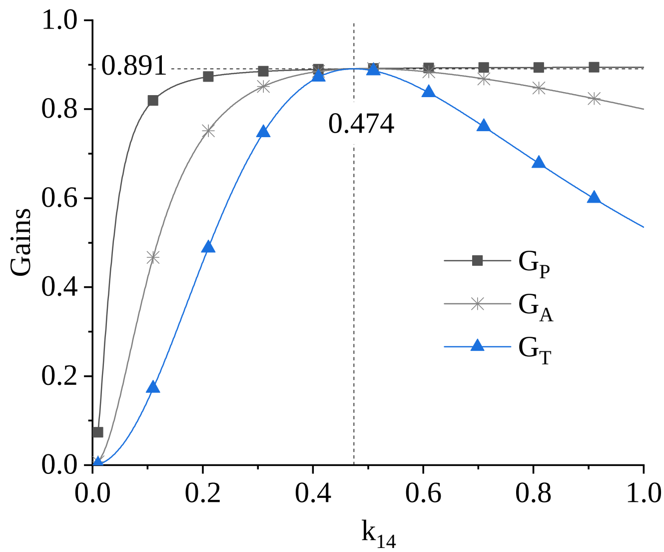

Figure 8 Results obtained for the gains by varying k_{14} when $k_{32} = k_{32}^{GT} = 0.539$. The termination impedances are G_S =20.0 mS and G_L =10.0 mS.

6 | CONCLUSION

A capacitive wireless power transfer system with two repeater elements allows for better performance with regard to efficiency versus distance than a system with none or a single repeater. We analytically solved two problems applying a network formalism.

First, the optimal terminations were determined for a two repeater system with given components and fixed couplings between the different resonators. It was found that in order to maximize the three main gains, the optimal values of the source

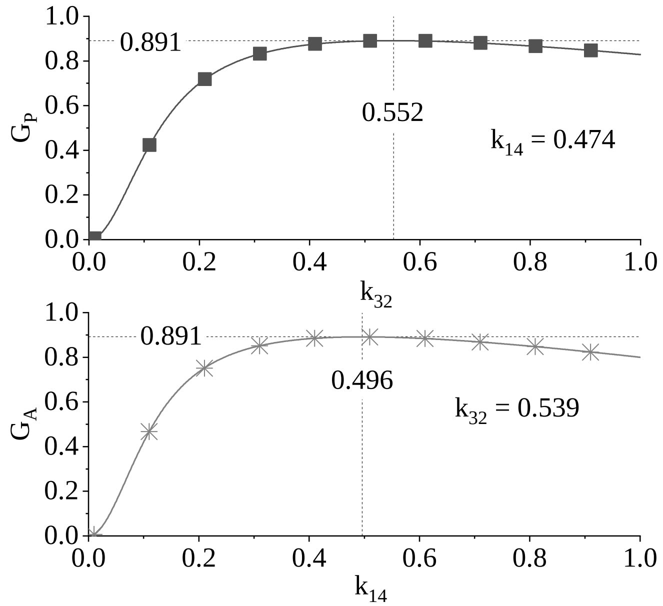

Figure 9 Results obtained for the power gain when $k_{14} = k_{14}^{GT} = 0.474$ and for the available gain when $k_{32} = k_{32}^{GT} = 0.539$. The termination impedances are G_S =20.0 mS and G_L =10.0 mS.

conductance G_S and load G_L must equal the conjugate image conductances. In this configuration, the ultimate gain G_M can be attained.

Second, the optimal couplings k_{14} and k_{32} were calculated for a system with given (non-optimal) terminations. By varying the couplings, impedance matching can be realized. However, it was found that the optimal coupling values are not the ones which realize a network with image conductances G_S and G_L. We demonstrated that:

- The power gain G_P is maximized with respect to k_{32} when the load admittance, as seen by the circuit formed by resonators 1, 4 and 3 equals its output image conductance.
- Analogous, the available gain G_A achieves a maximum with respect to k_{14} if the source admittance, as seen by the network formed by resonators 4, 3 and 2 equals its input image conductance.
- The maximum of the transducer gain G_T with respect to both couplings k_{14} and k_{32} is achieved when the source and load admittances, as seen by the circuit formed by the two repeaters, are equal to their respective image conductances.

Due to the resistive losses in transmitter and receiver, the ultimate gain can not be attained for non-optimal terminations, even when the couplings k_{14} and k_{32} can be optimized.

In conclusion, the two repeater CPT system can overcome the intrinsic limitation of a single repeater configuration which only has one degree of freedom, i.e., the coupling between the single repeater and either the transmitter or receiver. As a result, the single repeater system only allows for a single gain to be maximized at once. In contrast, the extra degree of freedom of the CPT system with two repeaters allows for the maximization of all three power gains.

Conflict of interest

The authors declare no potential conflict of interests.

Data availability statement

Data sharing is not applicable to this article as no datasets were generated or analyzed during the study.

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