Accepted for publication. DOI: 10.23919/EuMC54642.2022.9924407

Optimal Coupling for Capacitive Wireless Power Transfer with One Repeater

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***Abstract*— In order to extend the range of capacitive wireless power transfer, an electric field repeater between transmitter and receiver can be applied. In this work, a network formalism is adopted to analytically describe and maximize the system efficiency of a repeater setup. The resonator components, resistive losses, load and supply impedances are considered fixed and given. The efficiency is varied by acting on the coupling between transmitter, repeater and receiver only. It is found that it is only possible to optimize the system by varying the coupling between repeater and receiver while keeping the coupling between transmitter and repeater fixed. A condition, dependent on the value of the load, is determined for which the maximization solution exists. The analytical solution is verified through circuital simulations.**
***Keywords*— capacitive wireless power transfer, power gain, relay, resonator, repeater.**

I. Introduction

Near-field wireless power transfer (WPT) applies the electromagnetic field to power or charge an electrical device without the need for a physical cable between supply and load. If coils are used as transmitter and receiver, the *magnetic* field acts as transfer medium. This technology is called inductive power transfer (IPT) and is already commercially available on the market. Another WPT technology applies conductive plates to generate an *electric* field as transfer medium: capacitive power transfer (CPT). Compared to IPT, it has several advantages, the most important being a lower system cost and weight, lower heat dissipation and a larger tolerance to misalignment between transmitter and receiver. CPT has not yet entered the commercial market due to some hurdles, e.g., higher switching frequencies and operating voltages than IPT [1].

The main cause for the drawbacks of CPT compared to IPT is the lower power density for long distances due to the low coupling capacitance, usually in the picofarad range [1]. One way to mitigate this disadvantage, is by applying an electric field repeater to bridge longer distances (Fig. 1) [1], [2]. It is comparable to the use of relays for IPT applications [3], [4], [5]. By inserting an additional resonator circuit between the transmitter and receiver, the range of the WPT can be extended.

The question arises how one can optimize a CPT system with one repeater towards efficiency (power gain). Maximizing the power conversion efficiency corresponds to minimizing the power reflections at the output in order to optimize the transfer of power available at the output port to the load. In our previous work [6], the gains of the system were maximized by acting on the input and output terminations of the ports, i.e., by choosing the optimal source and load impedances. The system itself (including the couplings between transmitter, repeater and receiver) were considered fixed and given. However, in a practical WPT situation, the terminations of the system are often not free to choose and are imposed by the charging setup. Also the components of the CPT system itself are usually fixed. However, it is possible to act on the couplings of the system by changing the position of the electric field repeater with regard to the transmitter and/or receiver. In this work, the efficiency maximization problem is solved for a CPT system with fixed load by changing the couplings between the different resonators. This has already been done for IPT [5], but to our knowledge, no solution exists for the CPT technology.

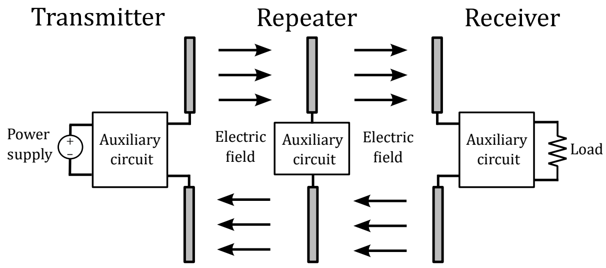

Fig. 1. A capacitive wireless power transfer system with one repeater: an additional resonator circuit between transmitter and receiver extends the range of the wireless power transfer from supply to load.

II. Capacitive wireless power system with one repeater

A. Problem description

The equivalent circuit of a basic CPT system with one repeater is depicted in Fig. 2. The power supply is represented by a sinusoidal current source I_S with internal shunt conductance G_S and angular frequency ω_0. The load is considered purely resistive and is represented by the conductance G_L. The subscripts 1 and 2 are applied for the transmitter and receiver respectively. The repeater, denoted by subscript 3, is placed between the transmitter and receiver.

The wireless links can be depicted by coupled capacitors C_i (i =1,2,3) [7]. The coupling strength between

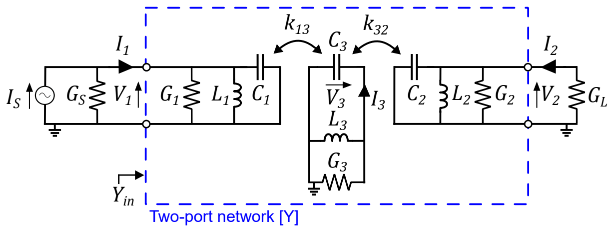

Fig. 2. An equivalent circuit representation of a capacitive wireless power transfer system with one repeater.

transmitter and repeater is expressed by the coupling factor k_{13}, defined as:

$$k_{13} = \frac{C_{13}}{\sqrt{C_1 C_3}} \tag{1}$$

with C_{13} the mutual capacitance between transmitter and repeater. Similarly, the coupling between repeater and receiver is k_{32}. In practical situations, the coupling between transmitter and receiver for a CPT system with a repeater is small compared to the other couplings, and can be neglected (i.e., k_{12} =0).

The resistive losses within the system are represented by the shunt conductances G_i. The input admittance $Y_{in} = G_{in} + jB_{in}$ of the two-port network is indicated in Fig. 2. It is assumed that the three resonators (transmitter, receiver and repeater) are synchronous and resonate at the same angular frequency ω_0. This can be realized by imposing (i =1,2,3):

$$L_i = \frac{1}{\omega_0^2 C_i}. \tag{2}$$

In this work, the following problem is considered and solved: how can one optimize the power gain (efficiency) of the system by acting on the couplings? For example, is it possible to optimize the system toward efficiency by e.g., changing the distance between transmitter and repeater (i.e., varying k_{13}), or changing the coupling between repeater and receiver (k_{32})? In a practical situation, data communication between the resonators may be required to optimize the couplings. It is assumed that the terminations of the system (i.e., the load and generator conductances) are given and fixed, as well as the components of the CPT system (represented by the lumped conductances G_i, inductances L_i and capacitors C_i.).

B. Admittance matrix

The CPT system can be considered as a two-port network, with the supply at the input port and the load at the output port (Fig. 2). The two-port network can be fully characterized by its admittance matrix $\boldsymbol{Y}$, for which:

$$\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} y_{11} & y_{12} \\ y_{21} & y_{22} \end{bmatrix} \cdot \begin{bmatrix} V_1 \\ V_2 \end{bmatrix}. \tag{3}$$

At the operating angular frequency ω_0, the relations between the voltages and currents (as defined in Fig. 2) are given by:

$$I_1 = G_1 V_1 + j\omega_0 C_{13} V_3 \tag{4}$$

$$I_2 = G_2 V_2 + j\omega_0 C_{23} V_3 \tag{5}$$

$$0 = G_3 V_3 + j\omega_0 C_{13} V_1 + j\omega_0 C_{23} V_2. \tag{6}$$

These equations result into the admittance matrix $\boldsymbol{Y}$:

$$\boldsymbol{Y} = \frac{1}{G_3} \begin{bmatrix} G_1 G_3 + \omega_0^2 C_{13}^2 & \omega_0^2 C_{13} C_{23} \\ \omega_0^2 C_{13} C_{23} & G_2 G_3 + \omega_0^2 C_{23}^2 \end{bmatrix}. \tag{7}$$

Table 1. Definition of the parameters adopted for the analysis.

$B_0 = \omega_0 C_2$	$Q_i = \frac{\omega_0 C_i}{G_i}$	$Q_L = \frac{\omega_0 C_2}{G_L}$
$n_{12} = \sqrt{\frac{C_1}{C_2}}$	$Q_{2T} = \frac{Q_2 Q_L}{Q_2 + Q_L}$	

The following parameters are defined (Table 1):

- The unloaded quality factor of the resonators is given by Q_i.
- The external quality factor of the receiver is Q_L.
- The loaded quality factors are indicated by Q_{1T} and Q_{2T}.
- The system is normalized on the susceptance B_0 of the receiver's main capacitance.
- The parameter n_{12} specifies the transformation ratio.

With these definitions, the elements of the admittance matrix (7) can be expressed as:

$$y_{11} = g_{11} = \frac{B_0 n_{12}^2}{Q_1}(1 + Q_1 Q_3 k_{13}^2) \tag{8}$$

$$y_{12} = y_{21} = g_{12} = B_0 n_{12} Q_3 k_{13} k_{32} \tag{9}$$

$$y_{22} = g_{22} = \frac{B_0}{Q_2}(1 + Q_2 Q_3 k_{32}^2). \tag{10}$$

C. Power gain

The power gain G_P or system efficiency is defined as the ratio of the power dissipated in the load to the power that the generator delivers to the two-port network. It can be expressed as function of the elements of the admittance matrix $\boldsymbol{Y}$ of the two-port network [6]:

$$G_P = \frac{G_L}{G_{in}} \left| \frac{y_{21}}{y_{22} + Y_L} \right|^2 \tag{11}$$

with G_{in} the real part of the input admittance Y_{in}, given by:

$$Y_{in} = \frac{I_1}{V_1} = y_{11} - \frac{y_{12} y_{21}}{y_{22} + Y_L}. \tag{12}$$

Taken into account the expressions for the admittance matrix elements (8), (9), and (10), the power gain G_P can be derived:

$$G_P = \frac{Q_1 Q_3^2 Q_{2T}^2 k_{13}^2 k_{32}^2}{(1 + Q_3 Q_{2T} k_{32}^2)(1 + Q_1 Q_3 k_{13}^2 + Q_3 Q_{2T} k_{32}^2) Q_L}. \tag{13}$$

III. Power Gain Optimization

By acting on the couplings between on the one hand the repeater and on the other hand the receiver and the transmitter, respectively, the power gain of the system can be varied. In this section, it is determined how the power gain can be maximized by acting on these couplings.

A. Optimizing the coupling between repeater and receiver

First, the coupling factor k_{13} between the transmitter and the repeater is considered fixed and is, for example, determined by the distance to be covered by the wireless link. The system efficiency or power gain G_P can be changed by varying the coupling k_{32} between repeater and receiver.

Deriving (13) with respect to k_{32} and equating to zero results in the value of k_{32}^{GP} that maximizes the power gain:

$$k_{32}^{GP} = \frac{\sqrt[4]{1 + Q_1 Q_3 k_{13}^2}}{\sqrt{Q_{2T} Q_3}}. \tag{14}$$

For this optimal value $k_{32} = k_{32}^{GP}$, the power gain results into:

$$G_{P32}^{M} = \frac{(1 - \sqrt{1 + Q_1 Q_3 k_{13}^2})^2}{Q_1 Q_3 k_{13}^2} . \frac{Q_{2T}}{Q_L}. \tag{15}$$

Since the maximum value of the coupling factor is unity, a solution is only physically possible under the condition that:

$$\frac{\sqrt{1 + Q_1 Q_3 k_{13}^2}}{Q_3} < Q_{2T}. \tag{16}$$

In case the condition above is not fulfilled, the power gain rises monotonically with k_{32}, i.e., the higher the coupling, the higher the efficiency. A maximum G_P is achieved for the ideal solution $k_{32} = 1$. Practically, this solution is not useful since it corresponds to shortening the distance to zero between repeater and receiver.

B. Optimizing the coupling between transmitter and repeater

Now, the coupling k_{32} between the repeater and the receiver is considered fixed and the coupling factor k_{13} between transmitter and repeater can be varied. However, deriving (13) with respect to k_{13} and equating to zero results in only one solution: $k_{13} = 0$. The power gain keeps increasing as k_{13} rises and no useful solution exists.

IV. Discussion

The analysis in the previous section has shown that:

- It is possible to optimize a CPT system with one repeater and given load towards power gain by varying the coupling k_{32} between repeater and receiver while keeping the coupling k_{13} between transmitter and repeater fixed. However, such is only possible if condition (16) is fulfilled.
- A power gain maximization is not possible by varying the coupling k_{13} between transmitter and repeater if the coupling k_{32} between repeater and receiver is fixed.

It can be shown [8] that in (15) the first factor corresponds to the maximum value the power gain can attain for a CPT system without a repeater, i.e. the CPT system consisting of only the transmitter and the receiver. The second factor in (15) equals the efficiency of the repeater circuit. This implies that optimizing the power gain by varying the coupling k_{32} to its optimal value k_{32}^{GP} is equivalent to applying the repeater circuit as an impedance matching network.

Since

$$Q_{2T} = \frac{Q_2}{1 + \frac{Q_2}{Q_L}}, \tag{17}$$

condition (16) implies that the power gain G_P can only be maximized by acting on the coupling between repeater and receiver if Q_L is not too small. In other words, the efficiency can only be maximized if the resistive value of the load is smaller than a value which is dependent on k_{13}. The higher the resistance of the load, the lower the limit value of Q_{2T} for the existence of a solution. Thus, a too high value of k_{13} can make it impossible to reach the maximum value of the power gain G_P.

V. Numerical verification

The previous analytical results are verified by a numerical example that has been simulated by the AWR Design Environment circuit simulator.

A CPT system with one electric field repeater is considered. The configuration is described by its equivalent circuit (Fig. 2), with the numerical parameters summarized in Table 2. The operating frequency is f_0 equal to 10 MHz. The shunt inductors L_i are set so to make the capacitors C_i resonating at f_0. It can be calculated [6] that a maximum power gain is attained for a terminating load of 171 Ω. However, for this simulation, a non-optimal load of 50 Ω is chosen. From (2) and Table 1, the shunt inductances and quality factors are calculated (Table 3).

Table 2. Given parameters for the analyzed CPT system with one repeater.

Quantity	Value	Quantity	Value
G_1	1.50 mS	C_1	300 pF
G_2	1.00 mS	C_2	250 pF
G_3	2.00 mS	C_3	350 pF
C_{13}	150 pF	f_0	10.0 MHz
C_{23}	100 pF	$G_L(R_L)$	20 mS (50 Ω)
k_{13}	0.2		

Table 3. Calculated parameter values for the simulated CPT system with one repeater.

Quantity	Value	Quantity	Value
L_1	0.844 μH	Q_1	12.6
L_2	1.013 μH	Q_2	15.7
L_3	0.724 μH	Q_3	11.0
n_{12}	1.095	Q_{2T}	0.748
B_0	15.7 mS	Q_L	0.785
k_{32}^{GP}	0.557	G_{P32}^{M}	41.7 %

The coupling factors can be varied in order to maximize G_P. Figure 3 depicts (13): the power gain G_P as function of the coupling factors k_{13} and k_{32}. It can be observed that the power gain keeps increasing as k_{13} rises for fixed k_{32}. However, for a given k_{13}, an optimal value different from unity exists for k_{32}. Note that for low values of k_{32}, G_P is nearly independent of k_{13}. Low values of the power gain are obtained, even for high coupling factor k_{13}.

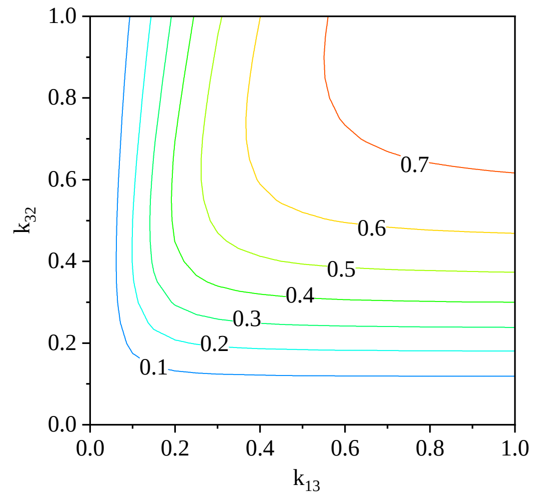

Fig. 3. Power gain as function of k_{13} and k_{32}.

Condition (16) indicates that a too high value of k_{13} can make it impossible to maximize the power gain. This is confirmed by the simulation: for $k_{13} \geq 0.7$, G_P continues to grow as k_{32} increases.

The coupling factor k_{13} between transmitter and repeater is chosen to be fixed at 0.2 in order to allow for a solution at a load of R_L=50 Ω. Figure 4 depicts the power gain as function of varying k_{32}. A maximum power gain G_{P32}^{M} of 41.7 % is attained at k_{32}^{GP}=0.557, corresponding to the analytical derivation from (14) and (15).

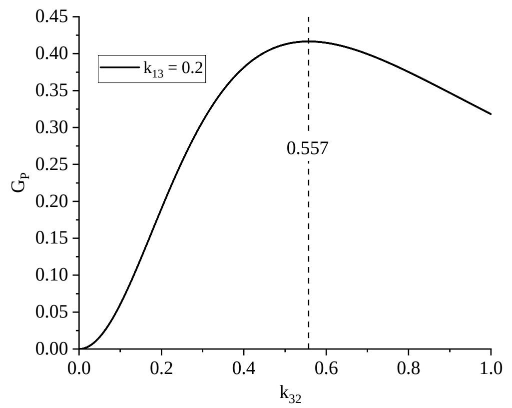

Fig. 4. Simulated power gain as function of k_{32}.

VI. Conclusion

A capacitive wireless power transfer system between one transmitter and one receiver coupled through an electric field repeater has been analyzed via a network formalism. Assumed all network elements fixed and given, the problem of maximizing the efficiency of the system by only acting on the couplings between the resonators had been analytically solved. First, it was shown that the power gain can be maximized by changing the coupling between repeater and receiver to an optimal value while keeping the coupling between transmitter and repeater fixed. However, a solution only exists if condition (16) has been met. Next, it was demonstrated that varying the coupling between transmitter and repeater does not render a solution. Finally, the analytical results were validated for a numerical example by circuital simulations.

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