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Maximizing the Power Transfer for a Mixed Inductive and Capacitive Wireless Power Transfer System

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***Abstract*—Nowadays, near-field wireless power transfer is realized by inductive or capacitive coupling. Power transmission is respectively accomplished by a time-varying magnetic or electric field as medium. Recently, mixed or hybrid near-field wireless power transfer is being developed as a possible mean to increase the power density of the system by utilizing both the magnetic and the electric near-field. The fundamental basics of mixed coupling are well understood. However, the implications of the mixed coupling theory on wireless power transfer applications is not rigorously described. Moreover, no general description is available that allows for a detailed comparison between current hybrid systems, especially for a series topology of inductive and capacitive coupling. In this work, we analytically solve a general mixed wireless power transfer configuration with series topology. We determine the optimal load to maximize the amount of power transfer and calculate the maximum achievable output power. The analytical derivation is validated by numerical simulation in SPICE. Our solution provides insight in the mixed wireless link and can serve as a reference point to evaluate the performance of mixed systems with regard to power transfer.**

***Index Terms*—capacitive power transmission, electromagnetic coupling, hybrid coupling, inductive power transmission, mixed coupling, mutual coupling, wireless power transmission.**

I. Introduction

Near-field wireless power transfer (WPT) across an air gap can be realized by coupled resonators. Two different principles exist:

- Inductive wireless power transfer (IPT) uses coupled inductors to transfer energy from a transmitter coil to a receiver coil. Power transfer is accomplished by a time-varying magnetic field as medium [1].
- Capacitive wireless power transfer (CPT) applies coupled capacitances for the power transfer from a transmitter plate to a receiver plate. The medium that realizes the power transfer is a time-varying electric field. CPT can be seen as the dual technique of IPT [2].

The IPT technology can be considered as relatively mature. Therefore, a large range of IPT applications have already entered the market [1]. Most known are portable electronics and household devices (e.g., electric toothbrushes, smartphones, smart watches and wearables). In the higher power range, IPT is applied to industrial automation (e.g., automated guided vehicles and robots) and electric vehicles.

The adaptation of CPT technology is lagging compared to IPT. Research and development efforts for CPT are increasing,

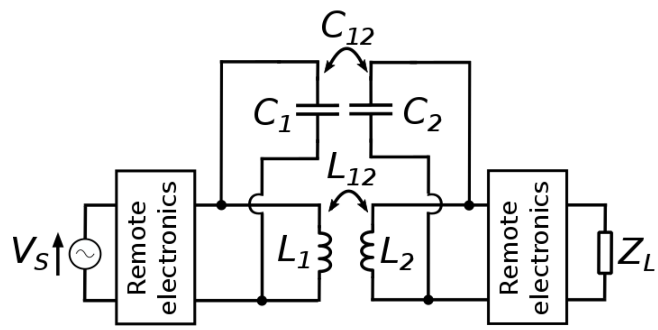

Fig. 1. Schematic circuit overview of a general mixed WPT system with IPT and CPT in parallel topology.

in particular for electric vehicles applications [3]. As far as we know, no commercial CPT applications are available on the market today.

Recently, mixed (also called hybrid) coupling is being considered by the research community as a possible mean to realize electromagnetic wireless power transfer [4]–[9]. Mixed coupling uses as well the magnetic as the electric near-field to transfer power. Both coupled inductors and capacitors are applied in transmitter and receiver. Consider for example Fig. 1. Power is wirelessly transferred from the voltage source V_S to the load Z_L by the coupled inductors L_1 and L_2 with mutual inductance L_{12}, *and* by the coupled capacitances C_1 and C_2 with mutual capacitance C_{12} [10]. Remote electronics (e.g., the compensation circuits, power converter and rectifier) are present in any practical WPT system to create resonance and improve the power transfer.

It is important to note that the capacitances C_1 and C_2, coupled by their mutual inductance C_{12} are an equivalent circuit model representation of reality. In practice, the capacitive coupling is realized by four plate electrodes: two at the transmitter's and two at the receiver's side. This set-up results in 6 capacitances: two main capacitances, created by each transmitter and receiver plate mutually, and four parasitic capacitances created between the different plates [10], [11]. The relation between those six physical capacitances and the equivalent circuit model is described in [11].

There are several arguments why a mixed or hybrid system could for certain applications be preferable to a single IPT or CPT system:

- It is possible to increase the amount of power transfer of an IPT or CPT system by enlarging the coil or capacitor plate area, respectively. However, in practical applications, this option is often not feasible due to size restrictions (e.g., smart watch, small sensor networks,...). Applying as well the magnetic as the electric field as medium for WPT allows for a larger amount of power transfer within the same space, or for a smaller transmitter and receiver area for the same amount of power transfer [4].

- In order to create a resonant circuit, different topologies are possible for an IPT system, depending on the application. However, all those topologies use at least one external capacitor. Analogous, any practical CPT system uses at least one external inductor to realize the resonance for WPT. Since any near-field WPT system applies at transmitter and receiver side at least both inductors and capacitances, it can be meaningful to apply mixed coupling to utilize the potential of both inductive and capacitive components [5], [6]. As a result, an increased power density for the entire WPT system can be achieved, fully using the circuit components and both magnetic and electric fields.
- Applying the different properties of magnetic and electric coupling, a mixed WPT system could allow for a coupling-independent regime at a fixed operating frequency [7]. In this way, the WPT system could become more robust against lateral misalignment and varying vertical displacements [8].

The fundamental basics of mixed coupling are well understood [10]. However, the implications of the mixed coupling theory on WPT applications is not rigorously described. Moreover, no general WPT description is available that allows for a detailed comparison between current hybrid systems. The scientific literature [4], [9], [10], [12] describes mixed WPT systems where IPT and CPT are arranged in a parallel topology as in Fig. 1. However, the experimental set-ups currently built in research labs do not apply the parallel topology, but a series topology as shown in Fig. 2. For example, Lu et al. [5] built a mixed 3.0 kW WPT system for the charging of electric vehicles. They apply a series topology of IPT and CPT to obtain a system efficiency of 94.5% at an operating frequency of 1 MHz. The same group also developed a 100 W prototype with a novel coupler structure [6] that allows for mixed WPT at an efficiency of 73.6%. Their coupler structure consists of a metallic grid and can be seen as an IPT and CPT coupling in series. Also Zhang et al. [7] applied the series topology for a distance-insensitive mixed WPT setup operating at efficiencies of 75% and higher.

The aforementioned examples study in detail their specific set-up, but these analysis are not applicable to a general mixed set-up. In this work, we rigorously solve a general mixed WPT configuration where the magnetic and electric couplings are arranged in series. Our solution provides insight in the mixed wireless link for WPT applications. More specifically, our contributions are the following:

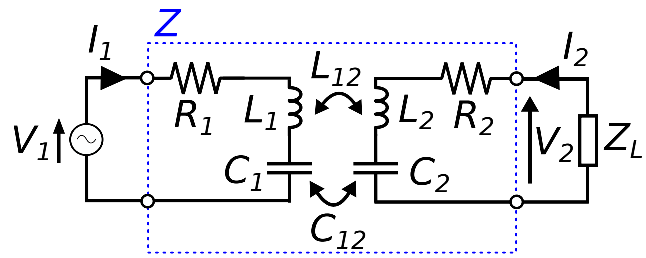

Fig. 2. Equivalent network of a mixed WPT setup, where IPT and CPT are arranged in series. The wireless link can be considered as a two-port network, characterized by its impedance matrix **Z**.

- The optimal load for a mixed WPT system with series topology is determined.
- The maximum achievable power transfer is calculated to serve as a reference point for evaluating the performance of mixed WPT systems.
- The analytical derivation is validated by numerical simulations in SPICE.

II. Methodology

A. Circuit analysis

Consider a general mixed WPT system, given by Fig. 2. Notice that the coupled inductors and coupled capacitors are arranged in series. The magnetic coupling factor k_m and electric coupling factor k_e are a measure for the strength of each coupling and are defined by [10]:

$$k_m = \frac{L_{12}}{\sqrt{L_1 L_2}} \tag{1}$$

$$k_e = \frac{C_{12}}{\sqrt{C_1 C_2}} \tag{2}$$

In a practical WPT system, the polarity of the coupled coils will be arranged in such a way that the IPT will enhance to the CPT. An opposite polarity could cancel out or decrease the CPT contribution. The WPT system is supplied by a voltage source with as peak-value voltage phasor V_1 and operating angular frequency ω_0. The resistive losses at transmitter and receiver side are represented by R_1 and R_2, respectively. The inductors and capacitors are chosen such that a resonant circuit is created:

$$\omega_0 = \frac{1}{\sqrt{L_1 C_1}} = \frac{1}{\sqrt{L_2 C_2}} \tag{3}$$

We consider the wireless link as a two-port network, fully characterized by its impedance matrix **Z**, given by:

$$\mathbf{Z} = \begin{bmatrix} R_1 + j\omega_0 L_1 + \frac{C_2}{j\omega_0\gamma_c} & j\omega_0 L_{12} + \frac{C_{12}}{j\omega_0\gamma_c} \\ j\omega_0 L_{12} + \frac{C_{12}}{j\omega_0\gamma_c} & R_2 + j\omega_0 L_2 + \frac{C_1}{j\omega_0\gamma_c} \end{bmatrix} \tag{4}$$

with

$$\gamma_c = C_1 C_2 - C_{12}^2 \tag{5}$$

The current-voltage relation of the two-port network is given by:

$$\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \mathbf{Z} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix} \tag{6}$$

with V_i and I_i (i =1,2) the peak voltage and current phasors, respectively, as defined on Fig. 2.

B. Power maximization

For the general mixed WPT circuit of Fig. 2, we determine analytically the optimal load that maximizes the power transfer. The power P_L dissipated by the load $Z_L = R_L + jX_L$ is given by:

$$P_L = \frac{|V_2|^2}{2R_L} \tag{7}$$

For convenience, we define the following parameters:

$$a_1 = \gamma_c L_1 \omega_0^2 - C_2 \tag{8}$$
$$a_{12} = \gamma_c L_{12} \omega_0^2 - C_{12} \tag{9}$$
$$a_2 = \gamma_c L_2 \omega_0^2 - C_1 \tag{10}$$
$$b = \gamma_c R_1 \omega_0 \tag{11}$$

In order to find the optimal load, we calculate the Thévenin equivalent as seen from the input port. The Thévenin voltage V_{th} is equal to the open circuit voltage at the output port of the two-port network. From (6), with $I_2 = 0$ and $V_2 = V_{th}$, we obtain:

$$V_{th} = \frac{a_1 + jb}{a_1^2 + b^2} a_{12} V_1 \tag{12}$$

The Thévenin impedance Z_{th} is equal to $Z_{th} = V_2/I_2$ for the two-port network with $V_1 = 0$. From (6), we find:

$$Z_{th} = \frac{a_{12}^2}{a_1^2 + b^2} R_1 + R_2 + j\frac{a_1 d + a_2 b}{\gamma_c \omega_0 (a_1^2 + b^2)} \tag{13}$$

with

$$d = (L_1 L_2 - L_{12}^2)\gamma_c^2 \omega_0^4 - (C_1 L_1 + C_2 L_2 - 2C_{12} L_{12})\gamma_c \omega_0^2 + \gamma_c \tag{14}$$

The load $Z_L' = R_L' + jX_L'$ that maximizes the power P_L is the complex conjugate of Z_{th}, according to the maximum power transfer theorem [13]. We obtain:

$$R_L' = \frac{a_{12}^2}{a_1^2 + b^2} R_1 + R_2 \tag{15}$$

$$X_L' = -\frac{a_1 d + a_2 b^2}{\gamma_c \omega_0 (a_1^2 + b^2)} \tag{16}$$

Notice that a non-zero reactance is necessary to maximize the power transfer to the load. The reactance X_L' compensates the complex part of the two-port network input impedance. Applying the load Z_L' results in a purely resistive input impedance at the input port.

TABLE I
SIMULATION VALUES.

Quantity	Value
f	100 kHz
V_1	1.00 V
R_1	1.00 Ω
R_2	1.00 Ω
L_1	100 μH
L_2	100 μH
L_{12}	20.0 μH
C_1	25.3 nF
C_2	25.3 nF
C_{12}	2.53 nF
k_m	20.0 %
k_e	10.0 %

A positive reactance X_L' corresponds to an inductor L_L', whereas a negative reactance X_L' corresponds with a capacitor C_L', with values:

$$L_L' = \frac{X_L'}{\omega_0} \tag{17}$$
$$C_L' = -\frac{1}{\omega_0 X_L'} \tag{18}$$

The maximum output power $P_{L,max}$ that a general mixed WPT system in series topology can achieve, can now be easily calculated. With the optimal load Z_L' applied to the two-port network, from (7) and the Thévenin circuit, we obtain:

$$P_{L,max} = \frac{a_{12}^2 |V_1|^2}{8R_1 a_{12}^2 + 8R_2(a_1^2 + b^2)} \tag{19}$$

This expression can serve as a reference point to evaluate the performance of specific mixed WPT systems with regard to power transfer and allows comparison between different systems.

III. NUMERICAL VALIDATION

We validate the analytical calculation by circuit simulation in SPICE. We consider the circuit of Fig. 2, assuming the numerical values given as in Table I.

Calculation from (15) and (16) results in an optimal load $Z_L' = R_L' + jX_L'$ for maximizing the power transfer of the system. We find $R_L' = 28.7$ Ω and $X_L' =$ -16.9 Ω. The negative imaginary part of the load corresponds with a capacitor C_L' equal to 94.2 nF. By applying this load to the mixed network, the maximum output power $P_{L,max}$ is achieved. From (19), we compute $P_{L,max} = 121$ mW.

First, we simulate the circuit in SPICE for varying load resistance. Fig 3 shows the output power dissipated by the load. The load reactance is kept fixed at $C_L' = 94.2$ nF. We observe that the maximum output power is achieved at the analytically calculated optimal resistive load of $R_L' = 28.7$ Ω, and corresponds with the calculated value of 121 mW.

Secondly, we perform the simulation for varying load reactance. Now, we keep the resistive part of the load fixed at its optimal value of $R_L' = 28.7$ Ω. Fig 4 shows the result. Again,

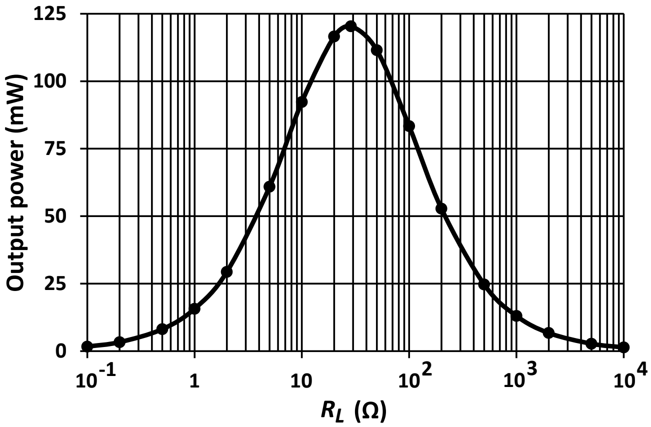

Fig. 3. The simulated output power for varying resistive load R_L. The load reactance remains fixed at $C'_L = 94.2$ nF.

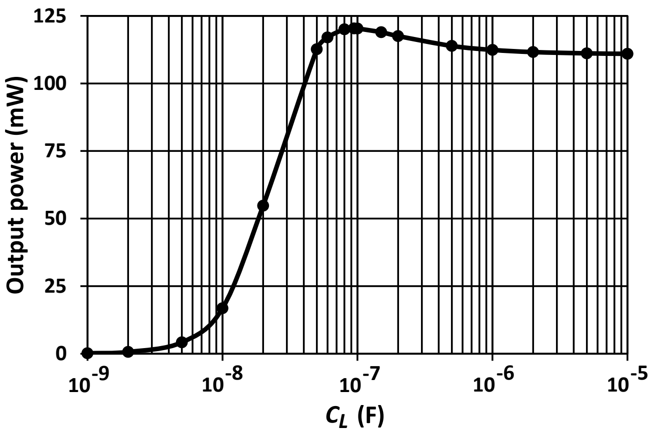

Fig. 4. The simulated output power for varying reactive load, i.e. varying capacitor C_L. The load resistance remains fixed at $R'_L = 28.7$ Ω.

perfect correspondence between simulation and the analytical derivation can be found: a maximum power transfer occurs at a capacitance of $C'_L = 94.2$ nF. Notice that for this example, the output power remains high for capacitances larger than C'_L.

IV. Conclusion

We determined the optimal load for a mixed WPT system with series topology. We found that a non-zero reactance is necessary to compensate for the complex input impedance of the system. The maximum achievable power transfer was calculated to serve as a reference point for evaluating the performance of mixed WPT systems. The analytical calculations were numerically validated by SPICE simulations.

Our solution can be seen as a first step towards a rigorous description of the coupling theory for mixed WPT applications. However, meticulous experiments on these systems are necessary to confirm whether or not this simplified model is an accurate description of reality, in particular with regard to the representation of the coupled capacitors in series with the coupled inductors.

References

[1] X. Lu, P. Wang, D. Niyato, D. I. Kim, and Z. Han, "Wireless charging technologies: Fundamentals, standards, and network applications," *IEEE Communications Surveys & Tutorials*, vol. 18, no. 2, pp. 1413–1452, 2016.

[2] B. Minnaert and N. Stevens, "Conjugate image theory applied on capacitive wireless power transfer," *Energies*, vol. 10, no. 1, p. 46, 2017.

[3] F. Lu, H. Zhang, and C. Mi, "A review on the recent development of capacitive wireless power transfer technology," *Energies*, vol. 10, no. 11, p. 1752, 2017.

[4] X. Chen, S. Yu, and X. Yang, "Hybrid wireless power transfer," *IECON 2017 - 43rd Annual Conference of the IEEE Industrial Electronics Society*, pp. 5348–5352, 2017.

[5] F. Lu, H. Zhang, H. Hofmann, and C. C. Mi, "An inductive and capacitive combined wireless power transfer system with LC-compensated topology," *IEEE Transactions on Power Electronics*, vol. 31, no. 12, pp. 8471–8482, 2016.

[6] F. Lu, H. Zhang, H. Hofmann, and C. C. Mi, "An inductive and capacitive integrated coupler and its LCL compensation circuit design for wireless power transfer," *IEEE Transactions on Industry Applications*, vol. 53, no. 5, pp. 4903–4913, 2017.

[7] X. Y. Zhang, C.-D. Xue, and J.-K. Lin, "Distance-insensitive wireless power transfer using mixed electric and magnetic coupling for frequency splitting suppression," *IEEE Transactions on Microwave Theory and Techniques*, vol. 65, no. 11, pp. 4307–4316, 2017.

[8] D. Vincent, P. H. Sang, and S. S. Williamson, "Feasibility study of hybrid inductive and capacitive wireless power transfer for future transportation," in *Transportation Electrification Conference and Expo (ITEC), 2017 IEEE*. IEEE, 2017, pp. 229–233.

[9] B. Minnaert and N. Stevens, "Conjugate image theory for non-symmetric inductive, capacitive and mixed coupling," in *Wireless Power Transfer Conference (WPTC), 2017 IEEE*. IEEE, 2017, pp. 1–4.

[10] J.-S. G. Hong and M. J. Lancaster, *Microstrip filters for RF/microwave applications*. John Wiley & Sons, 2004, vol. 167.

[11] L. Huang and A. P. Hu, "Defining the mutual coupling of capacitive power transfer for wireless power transfer," *Electronics Letters*, vol. 51, no. 22, pp. 1806–1807, 2015.

[12] H. Hirayama, T. Amano, N. Kikuma, and K. Sakakibara, "Undesired emission and biological effect of open-end and short-end antennas for coupled-resonant wireless power transfer," in *Electromagnetic Compatibility (APEMC), 2013 Asia-Pacific Symposium on*. IEEE, 2013, pp. 1–4.

[13] A. Bhaya, E. A. Herrera, and O. Diene, "Revisiting the maximum power transfer for linear n-ports with uncoupled loads and applications to power systems," *International Journal of Circuit Theory and Applications*, vol. 44, no. 9, pp. 1631–1656, 2016.