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Evaluation of the Vertical Magnetic Field Generated by a Spiral Planar Coil

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***Abstract*—The study of the magnetic field generated by planar spiral coils is important in several widespread application domains. However, calculating this magnetic field distribution is complex and requires expensive computational electromagnetic (CEM) tools. In this paper, we reduce this challenge to the evaluation of a single integral for the vertical field component for a general, planar spiral coil. We focus on the vertical field component due to its importance in most applications such as wireless power transfer. We verify our calculations by comparing them with the simulation results from the CEM tool 'CST EM Studio'. Our results make it possible to quickly determine this field with any standard mathematical software package, thus without the need for an expensive CEM tool.**

I. Introduction

The study of the magnetic field generated by planar spiral coils is important in several widespread applications. For example, spiral coils are used for inductive magnetic coupling, which has gained significant importance as an enabling technology in the field of contemporary electronic tools. They enable electronic circuits to transfer energy and information over a short-range link. Their main applications include Radio Frequency IDentification (RFID), Near Field Communication (NFC) and inductive wireless power transfer [1]-[2].

But also in power electronics, robotics, transformers and DC/DC converters, spiral coils are utilized [3]. Indeed, their flatness makes them a good replacement for an ordinary inductance for devices where thickness reduction is important [4]-[5]. Besides their flat configuration, the main advantage is their high inductance. Indeed, the greatest inductance can be obtained when using a spiral coil pattern [6].

Other technologies where planar spiral coils are used are NMR imaging [7], the detection of NQR signals [8], bioelectromagnetic experiments [9] and electromagnetic forming [10]. The latter is an impulse or high-speed forming technology using pulsed magnetic fields to apply Lorentz forces to workpieces without mechanical contact and without a working medium [11].

However, determining the magnetic field of this important spiral configuration is complex and forms an obstacle for a fast interpretation. It requires an expensive computational electromagnetic (CEM) tool to determine the magnetic field. Indeed, where it is possible to derive a closed-form mathematical expression for a circular current loop, this is not possible for a spiral current path [12]. Usually, the evaluation of the magnetic field induced by a spiral coil is either

- tackled by (expensive) electromagnetism finite element modelling, (e.g. [10]), or
- the spiral coil is approximated by concentric circles (e.g., [4],[13]).

In this paper, we evaluate the magnetic field strength generated by a spiral, thin wire, planar current path. Magnetic field evaluations have already been done for certain specific spirals, e.g., the Archimedean spiral [4] (Fig. 1) and the circle involute [14]. But to the best of our knowledge, this is the first work that determines a single integral for the magnetic field strength generated by *any* planar spiral. We focus on the vertical field component due to its importance in most applications, while the radial field component is less used for magnetic coupling purposes.

More specifically, the contributions of this paper are:

- A single integral expression for the vertical field component is constructed for a general, spiral planar coil (section II).
- we evaluate numerically this vertical field component (section III).
- we verify our calculation method by simulating the magnetic field distribution of a planar spiral in a CEM

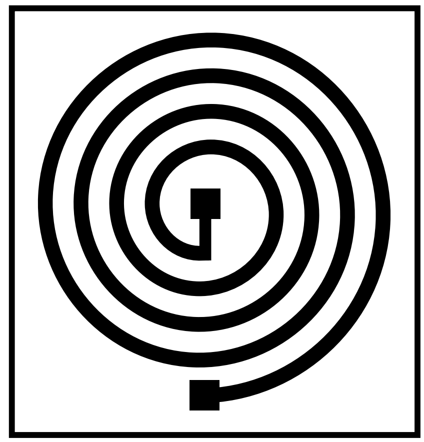

Fig. 1. Typical Archimedean spiral for use in miscellaneous electronic circuits.

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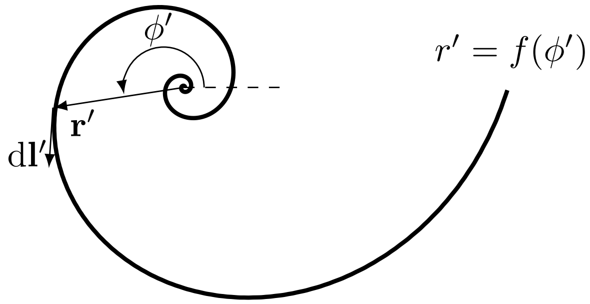

Fig. 2. The golden spiral ($b = 0.306349$) with ϕ between 0 and 6π.

tool and compare the simulation results with the results of our derivation (section III).

In this way, we make it possible to quickly determine the vertical magnetic field generated by a spiral planar coil. This can be done with any standard mathematical software package, thus without the need for an expensive CEM tool.

II. Theoretical development

The magnetic field strength $\mathrm{d}\mathbf{H}$ generated by a steady current I flowing in a filament of wire described by $\mathbf{r}'$, at the observation location $\mathbf{r}$ is described by the well-known Biot Savart equation [15]. Here, $\mathrm{d}\mathbf{l}'$ is the elementary displacement vector along the curve described by $\mathbf{r}'$.

$$\mathrm{d}\mathbf{H}\left(\mathbf{r},\mathbf{r}'\right) = \frac{I}{4\pi}\frac{\mathrm{d}\mathbf{l}' \times (\mathbf{r}-\mathbf{r}')}{\left|\mathbf{r}-\mathbf{r}'\right|^3} \tag{1}$$

The goal of this paper is to elaborate previous equation to a single integral in case of a planar spiral current path.

We chose the cylindrical coordinates system (r', ϕ', z') such that the planar spiral is located in the plane $z' = 0$. Then, a general planar spiral is defined by a parametric system, given by

$$r' = f(\phi') \tag{2}$$

where $f(\phi')$ is a monotonic, continuous function of the angle. An example is shown in Fig. 2, where part of the golden spiral ($f(\phi') = e^{b\phi'}$, with $b = 0.306349$) is plotted.

The displacement vector $\mathrm{d}\mathbf{l}'$ in cylindrical coordinates in the plane $z' = 0$ is given by [16]:

$$\mathrm{d}\mathbf{l}' = \mathrm{d}r'\mathbf{u}_{r'} + r'\mathrm{d}\phi'\,\mathbf{u}_{\phi'}. \tag{3}$$

where $\mathbf{u}'_r$ and $\mathbf{u}'_\phi$ are respectively the unit vector in the r' and ϕ' direction. Taken into account (2), we obtain:

$$\mathrm{d}\mathbf{l}' = \frac{\mathrm{d}f(\phi')}{\mathrm{d}\phi'}\mathrm{d}\phi'\,\mathbf{u}_{r'} + f(\phi')\mathrm{d}\phi'\,\mathbf{u}_{\phi'}. \tag{4}$$

The distance vector between an excitation location $\mathbf{r}'(f(\phi'), \phi', 0)$ and observation location $\mathbf{r}(r, \phi, z)$ is given by:

$$\begin{aligned}\mathbf{r}-\mathbf{r}' \;=\; & \left[r\cos\phi - f(\phi')\cos\phi'\right]\mathbf{u}_x + \\ & \left[r\sin\phi - f(\phi')\sin\phi'\right]\mathbf{u}_y + z\mathbf{u}_z\end{aligned} \tag{5}$$

and thus:

$$\left|\mathbf{r}-\mathbf{r}'\right|^2 = r^2 + f^2(\phi') - 2rf(\phi')\cos(\phi-\phi') + z^2 \tag{6}$$

In order to obtain the z-directed component of the magnetic field, we need $[\mathrm{d}\mathbf{l}' \times (\mathbf{r}-\mathbf{r}')]\cdot\mathbf{u}_z$. After some manipulations, one obtains that

$$\begin{aligned}&[\mathrm{d}\mathbf{l}' \times (\mathbf{r}-\mathbf{r}')]\cdot\mathbf{u}_z = \\ &\quad\left[r\frac{\mathrm{d}f(\phi')}{\mathrm{d}\phi'}\sin(\phi-\phi') - rf(\phi')\cos(\phi-\phi') + f^2(\phi')\right]\mathrm{d}\phi'\end{aligned} \tag{7}$$

Remark that there is no mathematical obstruction to elaborate the r and ϕ contributions, but we focus on the z-component due to its importance in inductively coupled planar coils. We obtain:

$$\begin{aligned}&\mathrm{d}\mathbf{H}\left(\mathbf{r},\mathbf{r}'\right)\cdot\mathbf{u}_z = \\ &\frac{I\left[r\frac{\mathrm{d}f(\phi')}{\mathrm{d}\phi'}\sin(\phi-\phi') - rf(\phi')\cos(\phi-\phi') + f^2(\phi')\right]}{4\pi[r^2+f^2(\phi')-2rf(\phi')\cos(\phi-\phi')+z^2]^{3/2}}\mathrm{d}\phi'\end{aligned} \tag{8}$$

and thus for the z-component of the magnetic field strength:

$$\begin{aligned}&H_z\left(\mathbf{r},\mathbf{r}'\right) = \\ &\frac{I}{4\pi}\int_{\phi'_1}^{\phi'_2}\frac{\left[r\frac{\mathrm{d}f(\phi')}{\mathrm{d}\phi'}\sin(\phi-\phi') - rf(\phi')\cos(\phi-\phi') + f^2(\phi')\right]}{[r^2+f^2(\phi')-2rf(\phi')\cos(\phi-\phi')+z^2]^{3/2}}\mathrm{d}\phi',\end{aligned} \tag{9}$$

where the current flows along the curve $r' = f(\phi')$ between ϕ'_1 and ϕ'_2.

Along the central axis of the spiral ($r = 0$), H_z becomes

$$\frac{I}{4\pi}\int_{\phi'_1}^{\phi'_2}\frac{f^2(\phi')}{[f^2(\phi')+z^2]^{3/2}}\mathrm{d}\phi' \tag{10}$$

In the center of the spiral ($r = z = 0$), H_z is simplified to

$$\frac{I}{4\pi}\int_{\phi'_1}^{\phi'_2}\frac{1}{f(\phi')}\mathrm{d}\phi' \tag{11}$$

III. Simulation and numerical verification

In order to verify the correctness of (9), we consider as an example the golden spiral of Fig. 2 with ϕ' between 0 and 6π (i.e. 3 revolutions) and with a steady current of 1 A. For this configuration, we simulate the magnetic field distribution with the CEM tool 'CST EM Studio' [17] and compare the simulation results with the numerical evaluation of (9).

We create within CST EM Studio a copper (annealed) wire with thickness 1 mm, following the configuration of Fig. 2. The wire is finite, starting in the center corresponding with $\phi' = 0$ and ending at a distance $r'_{max} = 322$ mm from the center, corresponding with $\phi' = 6\pi$. With current ports in CST, we force a current of 1 A through the wire, flowing from the center to the end of the spiral.

We use tetrahedral meshes to provide an explicit representation of the spiral geometry. With the stationary current solver, we compute the stationary current field. With these results as

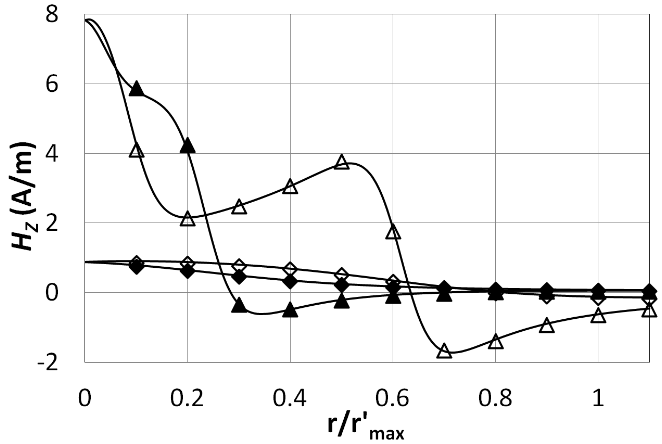

Fig. 3. The data points show the magnetic field H_z simulated with CST, at a height z of 0.1 (triangles) and 0.5 (diamonds) times r'_{max}, for $\phi' = \pi/2$ (filled markers) and $\phi' = 3\pi/2$ (open markers). The full line shows the corresponding calculation with MATLAB.

input, we use the magneto-static solver within CST EM Studio to calculate the magnetic field distribution.

The data points in Fig. 3 show the vertical component H_z of the magnetic field strength in A/m at a vertical height z of 0.1 and 0.5 times r'_{max}, for $\phi' = \pi/2$ and $\phi' = 3\pi/2$.

We now calculate equation (9) with the simulation environment MATLAB for the same heights z and same angles ϕ' as the simulation in CST. It is relevant to emphasize the differences between the MATLAB simulation and the numerical CST simulation. As opposed to the ideal, mathematical spiral in MATLAB, the spiral in CST:

- has a finite thickness of the wire (1 mm).
- does not consist of an ideal perfect conductor (annealed copper).
- is build out of different surface meshes, resulting in a different spatial resolution than the MATLAB calculation.

The full lines in Fig. 3 show the MATLAB calculation of equation (9). We clearly see that the numerical simulation in CST and the calculation in MATLAB are in excellent agreement with each other. This proves the correctness of equation (9) and of our evaluation of the vertical magnetic field generated by a spiral planar coil. At last, we want to remark that our analytical calculation is valid for *any* spiral, i.e. corresponding with (2), and not just for the golden spiral from our example.

IV. Conclusion

We determined a single integral for the vertical field component for a general, planar spiral coil. We simulated the magnetic field distribution in CST EM Studio and compared the simulation results with our calculation, proving its correctness. The proposed method delivers a simple numerical tool for the evaluation of the vertical magnetic field distribution generated by any spiral, planar coil.

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