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Efficiency Angle as Figure of Merit for Reciprocal MIMO Networks

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***Abstract*—The efficiency angle is originally only defined for single-input single-output power transfer systems. It acts as an angular figure of merit for the efficiency. However, multiple-input multiple-output power transfer systems are on the rise. In this work, the definition of the efficiency angle is extended to general reciprocal systems with multiple sources and loads. As a result, the efficiency angle can be applied to reciprocal systems with multiple transmitters and receivers; it facilitates the design, optimization and comparison of (wireless and wired) power transfer systems.**

I. INTRODUCTION

The goal of a power transfer system is to transport energy from a source to a load. The simplest setup consists of a single source and a single load: a single-input single-output (SISO) system. In the context of wireless power transfer (WPT), the source and load are often called the transmitter and receiver, respectively. The rise of (mainly inductive) WPT in the market for as well consumer as industrial applications has started an evolution towards the charging of several receivers at once by a single transmitter: a single-input multiple-output (SIMO) configuration. Moreover, an evolution towards multiple transmitters with one (MISO) or multiple (MIMO) receivers is expected.

In order to quantitatively characterize the performance of a (wireless or wired) power transfer system, and compare it quickly to alternatives, figure of merits can be used. These also function as design tool to evaluate the influence of one parameter change to the total performance.

One such figure of merit is the efficiency angle θ, which is related to the power gain (power conversion efficiency) of a power transfer system [1], [2]. It allows to determine the maximum possible efficiency of a system without needing to rigorously model or even know the internal structure of the system; simply determining (e.g., by measurement, theoretical calculation or via simulation) the input and output voltages and currents at the ports is sufficient. It allows for a lucid expression for the efficiency and is intuitively comprehensible by a graphical representation [1], [2].

However, the efficiency angle θ is originally only defined for SISO configurations [1]. It was recently extended for a WPT system with multiple receivers (SIMO) [3], and alluded to in a MIMO context, specifically for inductive WPT [4]. However, a explicit *general* all-encompassing definition for MIMO configurations is lacking.

In this work, the definition of the efficiency angle θ is extended to general reciprocal MIMO systems. Our goal is to emphasize that this figure of merit can be useful for the design of *any* reciprocal MIMO network (in particular with regard to power transfer systems), and not only to near-field WPT systems. It is applicable to all wireless links (e.g., optical, microwave, and acoustic WPT) and wired networks with multiple sources and/or loads, as long as the transfer system can be described (or approximated within the operating range) as a reciprocal multiport.

II. RECIPROCAL MIMO SYSTEM

Consider a reciprocal MIMO system with M input and N output ports (Fig. 1). An active input power P_{in} is delivered into the system via sources at the input ports. An active output power P_{out} is transferred to the M loads at the output ports. The power gain or efficiency η of the network is defined as:

$$\eta = \frac{P_{out}}{P_{in}} \tag{1}$$

Note that η is not only dependent on the MIMO network itself, but also on the values of applied loads at the output ports.

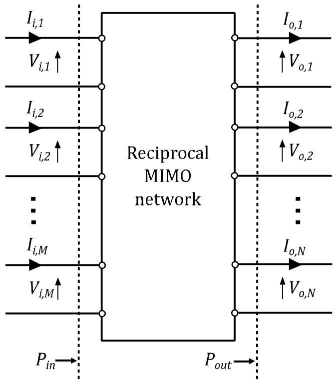

Fig. 1. A reciprocal multiport consisting of N input ports with voltages V_i and currents I_i, and M output ports with voltages V_o and currents I_o. The active input (P_{in}) and output power (P_{out}) are indicated.

Impedance matching at the output ports allows maximization of the efficiency η. It can be shown that the maximum attainable efficiency η_{max} of a fixed reciprocal multiport can be expressed as function of a single variable α and equals [4], [5]:

$$\eta_{max} = \frac{\sqrt{1-\alpha^2}-1}{\sqrt{1-\alpha^2}+1} = 1 - \frac{2}{1+\sqrt{1+\alpha^2}} \tag{2}$$

The figure of merit α is sometimes called the 'extended kQ-product', or in the context of near-field WPT simply 'the kQ-product' [6]. For a basic SISO inductive WPT system, consisting of a pair of coupled coils, the parameter α equals the unloaded kQ factor, i.e. the product of the coupling factor k, and the quality factor Q of the coupled coils. For a MISO or SIMO inductive or capacitive WPT system, it can be expressed as a system kQ product [3], [7]. We emphasize that this parameter is not only applicable on (inductive) WPT systems, but can be determined for any reciprocal multiport, even if no resonators are present.

III. THE EFFICIENCY ANGLE FOR MIMO SYSTEMS

We define the efficiency angle θ for MIMO systems in the range $0 < \theta < \pi/4$ rad as:

$$\tan 2\theta = \alpha \tag{3}$$

Figure 2 depicts the relationship between the extended kQ-product α and the efficiency angle θ in degrees; whereas the figure of merit α spans a few orders of magnitude for relevant efficiencies, θ has a much smaller range and is only defined from 0 to 45°.

Obviously, definition (3) reduces to the efficiency angle θ for SISO configurations if $M = N = 1$. However, although subtle, our definition differs from the original SISO definition. In previous works [1], [2], the efficiency angle θ is defined as function of the elements of the impedance or admittance matrix of a two-port network. Here, we define θ *only* from the extended kQ-product α. This has two advantages:

- It allows for a single, unambiguously definition, valid for all matrix representation (a.o., immittance and S-parameter representations) at once. In other words, no different definitions are required, depending on the chosen representation of the multiport, if any.
- It allows for a natural extension of the efficiency angle θ from SISO to MIMO systems and results in a single, lucid definition. Indeed, expressing θ as function of the elements of e.g., an impedance matrix of a (large) multiport is far from straightforward, and sometimes only numerically possible [4].

The definition (3) allows to express (2) compactly as:

$$\eta_{max} = \tan^2 \theta \tag{4}$$

Figure 3 plots the maximum efficiency η_{max} as function of efficiency angle θ: the higher the figure of merit, the higher the efficiency performance.

The efficiency angle θ can be determined via the extended kQ-factor α by (3), even without knowing the internal structure of the power transfer system; the current-voltage relations at the ports, or a matrix representation (e.g., an immittance matrix of the multiport) is sufficient. For example, in the excellent work of Duong et al. [4], the procedure is described for a MIMO inductive WPT system for which the impedance matrix is given. This procedure can be extended to any reciprocal MIMO network. Other examples for SIMO, MISO and MIMO systems can be found in e.g., [3], [5], [7].

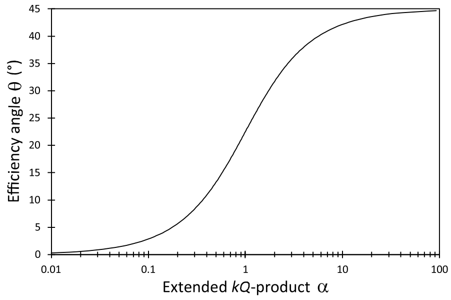

Fig. 2. The efficiency angle θ in degrees as function of the extended kQ-factor α.

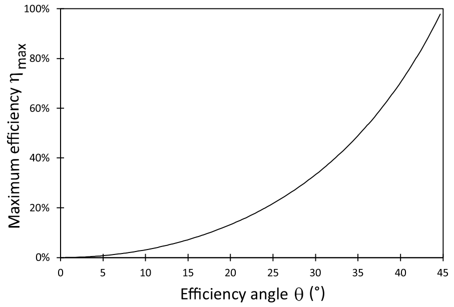

Fig. 3. The maximum efficiency η_{max} as function of the efficiency angle θ in degrees.

By extending the definition of the efficiency angle θ as figure of merit to MIMO configurations, it facilitates the design, optimization and comparison of power transfer systems with any number of transmitters or receivers.

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