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An Improved Algorithm for the Creation of Homogeneous Magnetic Field Distributions

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Abstract —
In inductive magnetic coupling, a set of two coils is used to transfer energy and/or information over a wireless, short-range link. In most cases, the generated magnetic field is not homogeneous, even though a homogeneous distribution would be advantageous for certain applications. Recently, an algorithm was proposed to calculate the distribution of the current traces of a planar coil to generate a homogeneous magnetic field. The algorithm gives excellent results, but is limited to about ten current traces because of instabilities of the algorithm. In this paper, we propose an improvement to the algorithm by using the generalized minimal residual method. Our improved numerical procedure eliminates the restriction on the number of traces and leads to a more accurate description of the desired current distribution in order to obtain a homogeneous vertical magnetic field.

1 INTRODUCTION

Inductive magnetic coupling has gained significant importance as an enabling technology in the field of contemporary electronic devices. The main applications include inductive wireless power transfer, Radio Frequency IDentification (RFID) and Near Field Communication (NFC). In these applications, the inductive coupling of a set of two coils is used to transfer energy and information over a wireless, short-range link. In wireless power applications, the emphasis is on the efficiency of energy transfer, with a minimalistic communication protocol (e.g., transmitting information on the load status of a battery at the receiver side) while for RFID or NFC, the communication aspect dominates.

Both approaches lead to different requirements on the coil set regarding e.g., the quality factor, but increased homogeneity of the magnetic field is a favorable, generic property for all these applications. For an inductive wireless power transfer system, a lateral displacement of the receiver coil to the transmitter coil changes the coupling factor and thus the efficiency of power transfer. A more homogeneous magnetic coupling enhances the flexibility of positioning, resulting in an improved efficiency. In the same way, a homogeneous magnetic coupling for RFID will increase the readout region and efficiency of information exchange.

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2 DESCRIPTION OF THE ALGORITHM

Recently, Waffenschmidt [1] proposed an algorithm to calculate the distribution of the current traces of a planar coil to generate a homogeneous magnetic field. In this way, it is possible to obtain a homogeneous magnetic coupling between planar coils with different diameters. Only the vertical component of the magnetic field is considered, because it is assumed that the axes of both coils are in line. The algorithm consists of two steps:

1. Find a current density distribution which generates a homogeneous magnetic field distribution.

2. Find a turn distribution for a coil which approximates this current density distribution, but with equal current in each turn.

Our suggested improvement of the algorithm is situated in the first part of the algorithm. We refer to [1] for a detailed description about the second part.

As in [1], we consider a disk shaped coil with radius R. We discretize this disk in N equally spaced current traces. The original algorithm, as Waffenschmidt himself correctly remarks in his paper, is limited to about $N = 10$ current traces because of numerical instabilities of the algorithm for more traces. Our improved algorithm eliminates the restriction on the number of traces.

Each current trace runs a current density J_i (with i from 1 to N) and contributes to the magnetic field at different positions. We consider N equidistant observation locations, all at the same height above the coil, and call H_i the magnetic field strength at each location. We can write:

$$\mathbf{H} = A \cdot \mathbf{J} \tag{1}$$

or in more detail:

$$\begin{bmatrix} H_1 \\ H_2 \\ \vdots \\ H_N \end{bmatrix} = \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1N} \\ a_{21} & a_{22} & \cdots & a_{2N} \\ \vdots & \vdots & \ddots & \vdots \\ a_{N1} & a_{N2} & \cdots & a_{NN} \end{bmatrix} \cdot \begin{bmatrix} J_1 \\ J_2 \\ \vdots \\ J_N \end{bmatrix} \tag{2}$$

The coefficients of matrix A (in units m) indicate the contribution of each current trace to the magnetic field at the different positions. They can be calculated using the closed expression for the vertical magnetic field distribution $H_z(r, z, R)$ of a circular current loop with radius R [2]:

$$H_z(r,z,R) = \frac{I}{2\pi} \frac{1}{\sqrt{(r+R)^2+z^2}} \cdot \left[\mathcal{K}(k) - \frac{r^2-R^2+z^2}{(r-R)^2+z^2}\mathcal{E}(k)\right] \quad (3)$$

with

$$k = \frac{2\sqrt{rR}}{\sqrt{(r+R)^2+z^2}} \quad (4)$$

Here are r and z the cylindrical coordinates of the observation location with the origin of the coordinates system chosen in the center of the circular loop. $\mathcal{K}(k)$ and $\mathcal{E}(k)$ are the complete elliptic integrals of the first and second kind respectively.

To determine the coefficients of matrix A, we consider a current in each turn successively and calculate for this particular current the magnetic field to achieve one column of matrix A, one after the other [1]. Since the system is linear, we can normalize on a current of 1 A in each turn.

After the calculation of the matrix A, the current density distribution which generates a homogeneous magnetic field distribution can be obtained by inverting (1):

$$\mathbf{J} = A^{-1} \cdot \mathbf{H} \quad (5)$$

with $\mathbf{H}$ the vector of the required magnetic field distribution.

Determining the inverse of A is by default done by LU decomposition which factors a matrix as the product of a lower triangular and an upper triangular matrix [3]. It can be viewed as the matrix form of standard Gaussian elimination. The improvement of the algorithm we suggest consists of using the generalized minimal residual (GMRES) method [4] for solving the inverse magnetic field problem instead of the LU decomposition. The GMRES method generates a sequence of orthogonal vectors for solving non-symmetrical linear systems. In this way, no instabilities occur within the algorithm.

3 COMPARISON

We now demonstrate the efficiency of the improved algorithm by comparing the two methods with each other for a different number of current traces. As mentioned above, the algorithm consists of two parts: determining the current density distribution and the corresponding turn distribution of a coil. Because our suggested improvement only affects the first part, we will only focus on the current distribution to make a fair comparison of the algorithms.

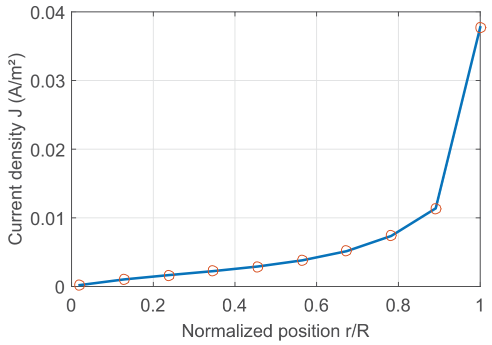

Figure 1: The current density calculated with LU decomposition (circles) and GMRES (solid line) for $N = 10$.

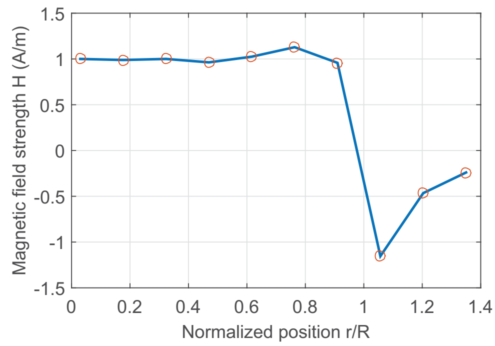

Figure 2: The magnetic field strength calculated with LU decomposition (circles) and GMRES (solid line) for $N = 10$.

We specified a required homogeneous, vertical magnetic field strength H_z of 1 A/m at a height above the coil of 5% of the coil radius R. For numerical stability, we only specify the magnetic field to 90% of the radius and not until the edge of the coil, just as in [1].

We perform the calculations with the software environment Matlab®. Fig. 1 shows the calculated current density with $N = 10$ for the original algorithm with LU decomposition (circles) and the improved algorithm with GMRES (drawn as a solid line for clearness of the figure). We immediately see that there is no difference between the methods for $N = 10$.

Fig. 2 shows the corresponding calculated magnetic field strength according to (1). We notice that

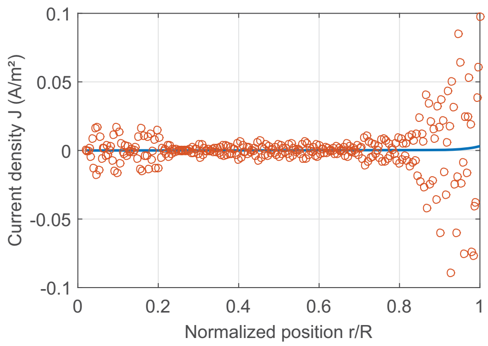

Figure 3: The current density calculated with LU decomposition (circles) and GMRES (solid line) for $N = 300$.

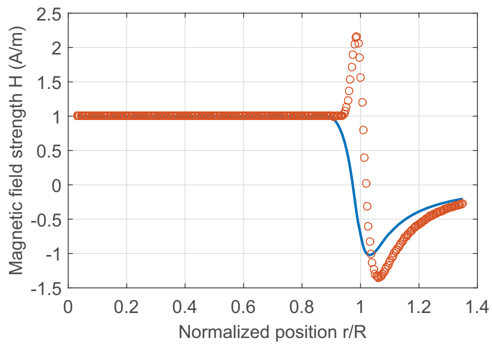

Figure 4: The magnetic field strength calculated with LU decomposition (circles) and GMRES (solid line) for $N = 300$.

choosing only 10 current traces leads to a quasi uniform field distribution. Remember that we specified a magnetic field strength of 1 A/m from the origin till 90% of the coil radius. In order to improve the homogeneity, more current traces and magnetic field evaluation positions are necessary.

Fig. 3 shows the calculated current density with LU decomposition and GMRES for $N = 300$. We immediately notice the numerous instabilities for the algorithm with LU decomposition, which are absent in the algorithm with GMRES. Also notice the undesirable higher ripple for the magnetic field strength with LU decomposition in Fig. 4. We want to remark that the computational time for this algorithm is less than 0.2 s on a standard computer (8 GB RAM, 2.6 GHz CPU), even for 300 current traces.

Another advantage of the GMRES algorithm is that it works better at higher heights above the coil. To illustrate this, we plot the current density for $N = 100$ at a height above the coil of 5% and 15% of the coil radius R (Fig. 5 and 6). We notice that also

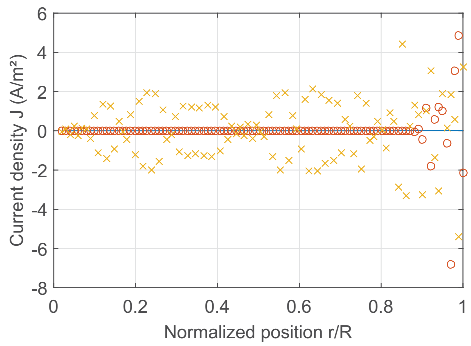

Figure 5: The current density calculated with LU decomposition for $N = 100$ at a height of 5% (circles) and 15% (crosses) of the coil radius R. The current density calculated with GMRES for $N = 100$ at a height of 5% of the coil radius R is shown as a solid line.

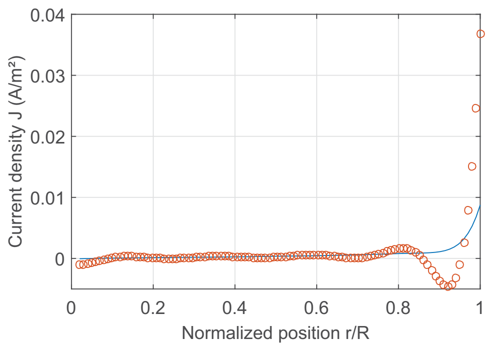

Figure 6: The current density calculated with GMRES for $N = 100$ at a height of 5% (solid line) and 15% (circles) of the coil radius R.

the GMRES algorithm starts to show instabilities at higher heights, but by far not as much as the algorithm with LU decomposition.

4 CONCLUSION

We proposed an improvement on an algorithm to calculate the distribution of the current traces of a planar coil to generate a homogeneous magnetic field. Our improved numerical procedure shows no instabilities at low heights, even for a large number of current traces. But also at higher heights, our method performs better. This algorithm leads to a more accurate description of the desired current distribution in order to obtain a homogeneous vertical magnetic field.

Acknowledgments

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